

參考用

Problem 1 - Ordinary Differential Equations (ODEs) (20%)

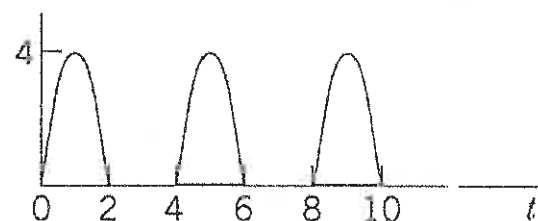
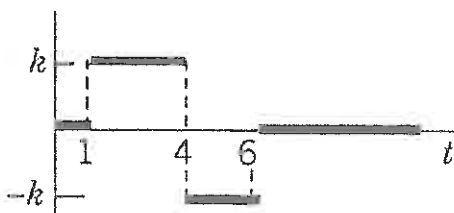
- (a) Find the solution for $xy' + 4y = 8x^4, y(1) = 2$ (6 %)
- (b) Solve $y'' + 4y' + (\pi^2 + 4)y = 0, y(\frac{1}{2}) = 1, y'(\frac{1}{2}) = -2$ (6%)
- (c) Find the solution to the 3rd-order ODE $x^3 y''' + xy' - y = x^2$. (8%)

Problem 2 - Power series & Frobenius methods (15%)

For a general **Legendre** equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ occurs in models exhibiting spherical symmetry. Please derive the recursion relation. (5 %) Please use power series method to solve it as $n=1$. (10%)

Problem 3 - Laplace transform (25%)

- (a) Please write down the unit step function of the following figures. (10%)



- (b) Solve $y'' + 2y' + 5y = 25t - 100\delta(t - \pi); y(0) = -2, y'(0) = 5$; δ is the direct delta function. (15%)

Problem 4 - Fourier series (10%)

Let $f(x) = x$ for $-\pi \leq x \leq \pi$. Please write the Fourier series of f on $[-\pi, \pi]$ and explain the Gibbs phenomenon.

注意:背面有試題

Problem 5 – Linear Algebra & Vector Calculus (20%)

- (a) Find eigenvalues and their corresponding eigenvectors of the matrix

$$C = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix} \quad (5\%)$$

- (b) For a square matrix
- A
- , show that
- A^{-1}
- exists if and only if the eigenvalues
- $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$
- are all nonzero, and then
- A^{-1}
- has the eigenvalues
- $1/\lambda_1, 1/\lambda_2, 1/\lambda_3, \dots, 1/\lambda_n$
- . (7%)

- (c) Find the surface integral of
- $\int_S \mathbf{F} \cdot \mathbf{n} \, dA$
- if
- $\mathbf{F} = [x^3 - y^3, y^3 - z^3, z^3 - x^3]$
- ,
- S
- : the surface of
- $x^2 + y^2 + z^2 \leq 25, z \geq 0$
- by the divergence theorem. (8%)

Problem 6 – Complex Analysis (10%)

$z = x + iy$ is a complex number and $f(z) = u(x, y) + iv(x, y)$ is a complex function.

- (a) Is the function
- $f(z) = e^x(\cos y - i \sin y)$
- an analytic function? Why? (3 %)

- (b)
- $f(z) = \cos\left[\frac{1}{2}\pi(1+i)\right]$
- , transfer
- $f(z)$
- in the form of
- $f(z) = u + iv$
- and find its functional value? (3 %)

- (c) Find all solutions for the complex function
- $\cosh z = -1$
- . (4 %)

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