

1. Find orthogonal trajectories of the following function

$$y = c/x^2.$$

(10%)

2. Solve the following initial value problems.

$$y'' + 2y' + 10y = 17\sin x - 37\sin 3x, \quad y(0) = 6.6 \quad y'(0) = -2.2$$

(15%)

3. Let  $A = \begin{bmatrix} 6 & -2 & -2 \\ 10 & -3 & 1 \\ -10 & 5 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 9 & 4 & -4 \\ 4 & 7 & 0 \\ -4 & 0 & 11 \end{bmatrix}$ , and  $a = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$ ,

please calculate  $AB$ ,  $A^T A$ ,  $(A^T)^2$ ,  $a^T A a$ ,  $a^T (A + A^T) a$

(15%)

4. Find a basis of eigenvectors and diagonalize the following matrix

$$\begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(15%)

5. Find the Fourier cosine series as well as Fourier sine series of the following function.

$$f(x) = \sin x, \quad (0 < x < \pi)$$

(15%)

6. Find eigenvalues and eigenfunctions of the following problem.

$$y'' + \lambda y = 0 \quad y(0) = y(1) \quad y'(0) = y'(1)$$

(15%)

7. Using Laplace transform, solve the following integral equation

$$y(t) + \int_0^t y(\tau) d\tau = 2$$

(15%)

參考用