

參考用

甲、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. For what value of the constant m is $f(x) = \begin{cases} \cos 3x, & x \leq 0 \\ mx, & x > 0 \end{cases}$ differentiable at $x = 0$?

Answer : _____

2. Find the horizontal asymptote of the graph of $y = x \tan(\frac{1}{x})$. Answer : _____.

3. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} (x-5)^n$.

Answer : _____.

4. The function $f(x) = e^x + x$, being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)$. Find the value of df^{-1}/dx at the point $f(\ln 2)$. Answer : _____

5. A passenger ship and an oil tanker left port sometime in the morning; the former headed north, and the latter headed east. At noon, the passenger ship was 40 miles from port and sailing at 30 mph, while the oil tanker was 30 miles from the port and sailing at 20 mph. How fast was the distance between the two ships changing at that time?

Answer : _____

6. Evaluate the integral. $\int_2^6 \frac{1}{x\sqrt{4x+1}} dx$. Answer : _____.

7. Find the volume under the surface $f(x, y) = e^{-x^2}$, bounded by the xz -plane and the planes $y = x$ and $x = 1$. Answer : _____.

8. Find the work done by the force $\mathbf{F} = xy\mathbf{i} + (y-x)\mathbf{j}$ over the straight line from $(1, 1)$ to $(2, 3)$. Answer : _____

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Suppose that x and y are related by the equation $x = \int_0^y \frac{1}{\sqrt{1+4t^2}} dt$. Show that d^2y/dx^2 is proportional to y and find the constant of proportionality.

2. Find the directional derivative of the function $h(x, y, z) = \cos xy + e^{yz} + \ln zx$ at $(1, 0, 1/2)$ in the direction of $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

3. Find the critical point of $f(x, y) = xy + 2x - \ln x^2 y$ in the open first quadrant ($x > 0, y > 0$) and show that f takes on a minimum there.