

1. Reduce to first order and solve

$$x^2 y'' + xy' - 4y = 0, \quad y_1 = x^2$$

(15%)

2. Solve the following initial value problem by Laplace transform

$$y'' + 2y' + 5y = 50t - 100, \quad y(2) = -4, \quad y'(2) = 14$$

(15%)

3. Show orthogonality on the given interval.

$$1, x, x^2 - \frac{1}{3}, x^3 - \frac{3}{5}x \quad -1 \leq x \leq 1$$

(15%)

4. Solve the following problem by Laplace transform,

$$\frac{\partial u}{\partial x} + 2x \frac{\partial u}{\partial t} = 2x, \quad u(x, 0) = 1, \quad u(0, t) = 1.$$

(15%)

5. Find the corresponding Fourier series of the following periodic function,

$$f(x) = \pi x^3 / 2, \quad -1 < x < 1, \quad f(x+2) = f(x)$$

(15%)

6. Let
- $A = \begin{bmatrix} 2 & -1 & 3 \\ -2 & 1 & 4 \\ 1 & 2 & -2 \end{bmatrix}$
- ,
- $B = \begin{bmatrix} -1 & 3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
- ,
- $\mathbf{a} = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}$
- , and
- $\mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$
- . Please

calculate  $AB^T$ ,  $(3A - 2B)^T \mathbf{a}$ ,  $\mathbf{a}^T A \mathbf{a}$ ,  $\mathbf{a} \cdot \mathbf{b}$ ,  $\det(AB)$ 

(15%)

7. Let
- $f = zy + yx$
- ,
- $\mathbf{v} = [y \ z \ 4z - x]$
- ,
- $\mathbf{w} = [y^2 \ z^2 \ x^2]$
- . Find

$$\begin{array}{lll} \text{a. } \nabla f & \text{b. } \nabla \cdot \mathbf{v} & \text{c. } \nabla \times \mathbf{w} \\ \text{d. } \nabla^2 f^2 & \text{e. } \nabla \cdot (\mathbf{v} \times \mathbf{w}) & \end{array}$$

(10%)

