

甲、計算、證明題：共 3 大題，每大題 12 分，共 36 分。請在答案卷上詳細寫出計算及證明過程，否則不予計分。

1. Let  $f(x, y) = \frac{x^3 + y^3}{x^2 + y^2}$  for  $(x, y) \neq (0, 0)$ . Is it possible to define  $f(0, 0)$  in a way that makes  $f$  continuous at the origin? Explain your answer and show your reasons. (Hint: Let  $x = r \cos \theta$ ,  $y = r \sin \theta$ )

2. Evaluate the line integral by two methods: (a) directly and (b) using Green's Theorem.

$$\oint_C x dx + y dy,$$

where  $C$  consists of the line segments from  $(0, 1)$  to  $(0, 0)$  and from  $(0, 0)$  to  $(1, 0)$  and the parabola  $y = 1 - x^2$  from  $(1, 0)$  to  $(0, 1)$ .

3. Determine if the given series converges or diverges. Explain your answer and show your reasons.

(a)  $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n}).$

(b)  $\sum_{n=2}^{\infty} \frac{2}{n \ln n}.$

乙、填充題：共 8 題，每題 8 分，共 64 分。請僅就答案部分依題號順序寫在答案卷上，不須呈現演算過程。

1. If  $\lim_{x \rightarrow -2} \frac{f(x)}{x^2} = 1$ , find  $\lim_{x \rightarrow -2} \left( f(x) + \frac{f(x)}{x} \right)$ . Answer : \_\_\_\_\_

2. Find  $\lim_{x \rightarrow 0} \frac{1}{3x^2} \int_{x^2}^0 \cos t dt$ . Answer : \_\_\_\_\_

3. Find the rate of change of pressure at the point  $P = (1, 2, 0)$  in the direction of  $\mathbf{i} + \mathbf{j} + \mathbf{k}$ , where the pressure is given by  $g(x, y, z) = xe^{-yz}$ . Answer : \_\_\_\_\_

4. If  $y = x^{\sin x}$ , find the derivative of  $y$  with respect to  $x$ . Answer : \_\_\_\_\_

5. Evaluate  $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^0 \frac{4}{1+x^2+y^2} dx dy$ . Answer : \_\_\_\_\_

6. Evaluate the double integral  $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$ . Answer : \_\_\_\_\_

7. Evaluate the definite integral  $\int_{-1}^1 \frac{e^{\tan^{-1} y}}{1+y^2} dy$ . Answer : \_\_\_\_\_

8. Find the work done by the force field  $\mathbf{F}(x, y) = e^{-y}\mathbf{i} - xe^{-y}\mathbf{j}$  in moving an object from  $P(0, 1)$  to  $Q(2, 0)$ . Answer : \_\_\_\_\_

參考用