

國立中央大學八十七學年度轉學生入學試題卷

地球科學系 三年級

科目：應用數學

共一頁 第一頁

50% 1. Solve the following initial value problems:

(1) $y'' + 2y' + 5y = 0$, $y(0) = 1$, $y'(0) = 5$.

(2) $y' + y \tan x = \sin 2x$, $y(0) = 1$.

(3) $y'' - 2y' + y = e^x + x$, $y(0) = 1$, $y'(0) = 0$.

(4) $x^2 y'' - 3xy' + 4y = 0$, $y(1) = -1$, $y'(1) = -1$.

(5) $y''' - 2y'' - y' + 2y = 0$, $y(0) = 3$, $y'(0) = 0$, $y''(0) = 3$.

10% 2. Let $f(t) = \sin^2 t + t \sin t$. Find the Laplace transform $\mathcal{L}\{f(t)\}$.

10% 3. Let the Laplace transform of $f(t)$ be $\mathcal{L}\{f(t)\} = \frac{2s}{s^2 + 2s + 5}$. Find $f(t)$.

10% 4. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

10% 5. Let a vector $\vec{V} = yz \vec{i} + 3zx \vec{j} + z^2 \vec{k}$.

Find (1) $\nabla \cdot \vec{V}$ and (2) $\nabla \times \vec{V}$.

10% 6. Find the Fourier series of the periodic function

$$f(x) = \begin{cases} -1 & \text{if } -\pi < x < 0 \\ 1 & \text{if } 0 < x < \pi \end{cases}, \quad f(x + 2\pi) = f(x).$$

參考用