國立中央大學八十五學年度轉學生入學試題卷

地球科學系　三年級　科目：應用數學　共 1 頁 第 1 頁

1. Solve the following differential equations (24%)

(1). $xy' + y + 3 = 0$.　(2). $y'' - 2y' + 10y = 0$, $y(0) = 4$, $y'(0) = 1$.

(3). $x^2y'' + 7xy' + 13y = 0$.　(4). $y'' - 2y' + y = e^x + x$.

2. Find a unit vector normal to the surface $x^2 + y^2 - z = 6$ at the point $(2, 3, 7)$. (4%)

3. Evaluate div $\vec{F}$ and curl $\vec{F}$, if $\vec{F} = (x + xz^2)\,\vec{i} + xy\,\vec{j} + yz\,\vec{k}$. (8%)

4. Show that $\vec{F} = 2xy\,\vec{i} + (x^2+1)\,\vec{j} + 6z^2\,\vec{k}$ is conservative , and find a scalar potential ϕ for it, i.e. grad $\phi = \vec{F}$. (8%)

5. Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ from $(0, 0, 0)$ to $(1, 2, 4)$ if $\vec{F} = x^2\,\vec{i} + y\,\vec{j} + (xz - y)\,\vec{k}$ along the curve given parametrically by $x = t^2$, $y = 2t$, $z = 4t^3$. (6%)

6. Evaluate $I = \iint_S (x^3\,dy\,dz + x^2y\,dz\,dx + x^2z\,dx\,dy)$, where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ $(0 \le z \le b)$ and the circular disks $z = 0$ and $z = b$ $(x^2 + y^2 \le a)$ (6%)

7. Find the Fourier series of the function $f(x)$, $f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ 0 & \text{if } x = 0 \\ k & \text{if } 0 < x < \pi \end{cases}$, $f(x + 2\pi) = f(x)$. (6%)

8. Find the Fourier transform of the function $f(x)$, $f(x) = \begin{cases} 1 & \text{if } -b < x < b \\ 0 & \text{otherwise} \end{cases}$. (6%)

9. (1). Find the inverse matrix A^{-1} of $A = \begin{pmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$. (8%)

(2). Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 0 & -1 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. (10%)

10. (1). Solve the equation $\ln z = \frac{1}{2}\pi i$, (2). Find the principal value of $(2i)^{\frac{1}{2}}$. (8%)

11. Integrate $\frac{z^2 + 1}{z^2 - 1}$ in the counterclockwise sense around a circle of radius 1 with center at the point $z_0 = 1$. (6%)