We use the symbol c for the speed of light, unless explicitly stated otherwise.

一、單選題 / 計有16題，每題5分，答錯或不作答不計分亦不扣分　　請於答案卡上作答

1. A particle has a total relativistic energy of 13 GeV and a relativistic momentum of 12 GeV/c. What is the rest mass of this particle?
(A) 0.25 GeV/c^2 (B) 1 GeV/c^2 (C) 2 GeV/c^2 (D) 5 GeV/c^2
(E) 15 GeV/c^2

2. A certain particle at rest lives for 1.2 ns. When the particle moves through the laboratory at a speed of $0.9c$, what is its lifetime according to an observer in the laboratory?
(A) 0.52 ns (B) 1.25 ns (C) 2.75 ns (D) 7.29 ns (E) 9.33 ns

3. One particle has mass m and a second particle has mass $m/3$. These two particles are moving at the same speed v directly toward one another along the x-axis. After the head-on collision, the particles merge to form only a new particle of mass M. What is the mass M of the resulting particle?
(A) $M = \frac{2m}{3}\sqrt{\frac{4-(v^2/c^2)}{1-(v^2/c^2)}}$ (B) $M = \frac{m}{3}\sqrt{\frac{4-(v^2/c^2)}{1-(v^2/c^2)}}$ (C) $M = \frac{2m}{3}\sqrt{\frac{1-(v^2/c^2)}{4-(v^2/c^2)}}$
(D) $M = \frac{m}{3}\sqrt{\frac{1-(v^2/c^2)}{4-(v^2/c^2)}}$ (E) $M = \frac{4m}{3}$

4. If the temperature of a blackbody doubles, (i) by what factor will the wavelength of maximum emission change? (ii) by what factor will the energy emitted off the surface each second change?
(A) 2, 16 (B) 1/2, 16 (C) 1/16, 16 (D) 1/16, 8 (E) 1/8, 8

5. Suppose that a 20 W lightbulb radiates primarily at a wavelength $\lambda \approx 1000\,\mathrm{nm}$. What is the number of photons emitted per second ? Choose the best answer that applies. (Planck's constant: $h = 6.63 \times 10^{-34}$ J·s).
(A) 3×10^{20} (B) 1×10^{20} (C) 3×10^{28} (D) 1×10^{28} (E) 1×10^{17}

6. A particle of mass m has the wave function
$$\psi(x) = a\,e^{ikx} + b\,e^{-ikx},$$
where a and b are complex constants and k is a real constant. The probability current density is equal to which of the following?
(A) 0 (B) $\hbar k/m$ (C) $\frac{\hbar k}{2m}\left(|a|^2 + |b|^2\right)$ (D) $\frac{\hbar k}{m}\left(|a|^2 - |b|^2\right)$ (E) $\frac{\hbar k}{2mi}\left(a^* b - b^* \alpha\right)$

7. A free particle with initial kinetic energy E and de Broglie wavelength λ enters a region in which it has potential energy V $(< E)$. What is the particle's new de Broglie wavelength?
(A) $\lambda(1 - V/E)$ (B) $\lambda(1 + E/V)$ (C) $\lambda(1 - V/E)^{-1/2}$ (D) $\lambda(1 + V/E)^{1/2}$
(E) $\lambda(1 - E/V)^{-1}$

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8. The energy eigenstates for a particle in an infinite square well have normalized wave functions ϕ_n and energies E_n, where $n = 1, 2, 3\cdots$. The particle is in a state described as follows:

$$\psi(x) = C\left[\phi_1(x) + \phi_2(x) + \phi_3(x)\right],$$

where C is the normalization constant. Which of the following is the average energy in this state?
(A) 0　(B) $E_1 + E_2 + E_3$　(C) $E_1^2 + E_2^2 + E_3^2$　(D) $3\,E_1$　(E) $14E_1/3$

9. Consider a one-dimensional quantum harmonic oscillator with potential energy $V(x) = m\omega^2 x^2/2$. At time $t = 0$ the state of the oscillator is described by a wave function that is a superposition of the ground state (ϕ_0) and first excited state (ϕ_1):

$$\Psi(x, 0) = C\left[\phi_0(x) + \phi_1(x)\right],$$

where C is the normalization constant, ϕ_0 and ϕ_1 are themselves normalized. Which of the following is the energy expectation value for the state at time $t > 0$?
(A) 0　(B) $\hbar\omega\cos(\omega t)$　(C) $\hbar\omega$　(D) $\hbar\omega e^{-i\omega t}$　(E) $\hbar\omega/2$

10. A particle of mass m and spin zero is in a three-dimensional isotropic well described by

$$V(r) = \frac{1}{2}m\omega^2 r^2, \quad \text{where } r^2 = x^2 + y^2 + z^2.$$

How many states have energy $7\hbar\omega/2$?
(A) 1　(B) 2　(C) 4　(D) 6　(E) 9

11. Which of the following is an allowed set of quantum numbers for an energy state before the hydrogen atom makes a spontaneous transition to the ground state ($n = 1, \ell = 0$)? n is the principal quantum number and ℓ is the orbital angular momentum quantum number.
(A) ($n = 2, \ell = 0$)　(B) ($n = 3, \ell = 1$)　(C) ($n = 3, \ell = 0$)　(D) ($n = 3, \ell = 2$)
(E) ($n = 2, \ell = 2$)

12. For the angular momentum quantum number $\ell = 2$, how many possible values are there for the magnetic quantum number m_ℓ?
(A) 1　(B) 2　(C) 3　(D) 4　(E) 5

13. The solution to the Schrödinger equation for the ground state of hydrogen is

$$\psi_0 = \frac{1}{\sqrt{\pi a_0^3}}\exp(-r/a_0),$$

where a_0 is the Bohr radius and r is the distance from the origin. Which of the following is the most probable value for r?
(A) 0　(B) $a_0/2$　(C) a_0　(D) $2a_0$　(E) ∞

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14. The energy eigenvalue for the $n = 1$ energy state of hydrogen is E_1. A magnetic field B is applied in a direction that is defined to be the z direction. The energy level splits into $E_1 \pm \Delta E$. What is ΔE? (m_e is the electron mass; e is the elementary charge.)
(A) $\frac{e\hbar B}{2m_e}$ (B) $\frac{e\hbar B}{m_e}$ (C) $\frac{2e\hbar B}{m_e}$ (D) $\frac{e\hbar}{2m_e}$ (E) $\frac{e\hbar}{m_e}$

15. Neutrinos are fermions. The mass of the neutrino is so close to zero that the energy can be approximated as $E = pc$, where p is the momentum and c is the speed of light. We assume that there can only be one neutrino in an energy state. What is the Fermi energy of a gas of N free neutrinos in a cube with sides of length L?
(A) $\hbar\pi c\left(\frac{6}{\pi}\frac{N}{L^3}\right)^{1/3}$ (B) $\hbar\pi c\left(\frac{6}{\pi}\frac{N}{L^3}\right)^{1/2}$ (C) $\hbar\pi c\left(\frac{6}{\pi}\frac{N}{L^3}\right)$ (D) $\hbar\pi c\left(\frac{6}{\pi}\frac{N}{L^3}\right)^{2/3}$
(E) $\hbar\pi c\left(\frac{6}{\pi}\frac{N}{L^3}\right)^{3/2}$

16. Consider a free-electron gas in three dimensions. What is the relation between the total energy (E_{tot}) and the Fermi energy (E_f)?
(A) $E_{\text{tot}} \propto E_f^2$ (B) $E_{\text{tot}} \propto E_f^{3/2}$ (C) $E_{\text{tot}} \propto E_f^{1/2}$ (D) $E_{\text{tot}} \propto E_f$
(E) $E_{\text{tot}} \propto E_f^{5/2}$

二、多選題 / 計有4題，每題5分，ABCDE每一選項單獨計分，答錯一個選項，倒扣 1 分，倒扣至本大題（即多選題）0分為止

17. Which statements are true regarding Compton scattering?
   (A) The wavelength of the scattered photon is shorter than that of the incoming photon.
   (B) Momentum is conserved in the collision.
   (C) Compton scattering reveals the particle nature of light.
   (D) It describes the collision between two photons.
   (E) The Compton-scattering formula is derived by using relativistic kinematics.

18. Which statements are true regarding photoelectric effect?
   (A) The maximum kinetic energy of the photoelectrons depends only on the frequency of the incident light.
   (B) The number of photoelectrons is proportional to the intensity of the incident light.
   (C) The stopping potential depends linearly on the frequency of the incident light.
   (D) A bright light yields photoelectrons of higher energies than a dim one of the same frequency.
   (E) This effect can be explained by the classical electromagnetic theory of light.

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19. Which statements about the semiconductors are true? An As atom has five valence electrons, A Si atom has four valence electrons and a B atom has three valence electrons.

    (A) When temperature increases, the electrical resistivity increases.

    (B) Holes act as if they were positively charged particles.

    (C) After doping the crystal lattice of Si atoms with As atoms, the material becomes an n-type semiconductor.

    (D) Semiconductors with excess holes are p-type semiconductor.

    (E) After doping the crystal lattice of Si atoms with B atoms, the material becomes an p-type semiconductor.

20. Which descriptions about the ground state of the Helium atom are correct?

    (A) The spin state is singlet.

    (B) The spin singlet and triplet states are degenerate.

    (C) The total spin is 1.

    (D) The orbital angular momentum is 1.

    (E) The overall wave function is antisymmetric.

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