1. Please provide simple answers for the following questions. **No need for explanations.** (a) Are photons bosons or fermions? (b) The energy of electron orbitals in an unperturbed hydrogen-like atom is determined by the quantum number n or l? (c) In quantum mechanics, can position and momentum of a particle be simultaneously determined ? Yes or no? (d) "The Hamiltonian H acting on the wave function Ψ which represents a real physics system lead to $H\Psi=E\Psi$, where E is the energy of this system." Is this statement true or false? (e) What is the eigenenergy of a simple harmonic oscillator? (5% each)
2. Explain the following subjects: (a) cosmic microwave background; (b) superconductors; (c) topological insulators; (d) gravitational wave. (5% each)
3. The wave function of a particle in a one-dimensional infinite square well of width a between x=0 and a at t=0 is given by $\Psi(x,t=0)=(2/7)^{1/2}\psi_1(x)+(5/7)^{1/2}\psi_2(x)$ where $\psi_1(x)$ and $\psi_2(x)$ are the ground state and the first excited stationary states. (a) Write down the wave function $\Psi(x,t)$. (10%) (b) You measure the energy of the particle at t=0. Write the possible values of the energy and the probability of measuring each. (10%) (c) Calculate the expectation value of the energy in the state $\Psi(x,t)$ above. (5%)
4. (a) Can the free particle wave function e^{ikx} be normalized in a box with size L?(10%) (b) When L is large, show that you can construct the (nearly) orthonormal eigenstates of the free particle from e^{ikx}. (10%)
5. In a metal, the group velocity of a conducting carrier is $v_k = \frac{d\omega}{dk}$, where $\varepsilon(k)$ is the dispersion relation or the band structure of the carrier. (Consider a simplified one dimensional case.) Show that $F = \hbar\frac{dk}{dt}$ by a semi-classical approach, where F is the experienced force on the carrier. (10%)