(倒扣至該大題0分為止)

單選題，答案請填於答案卡。一題五分，答錯倒扣一分，整題不作答不給分也不扣分。

1. For a 3-dimensional electron gas at low temperature, $kT \approx 0$, the Fermi energy E_F is proportional to $n^{2/3}$, where n is the number of electrons per unit volume. If the electrons are confined in a 2-dimensional area, the Fermi energy is related to the surface density of the electrons σ by $E_F = const \cdot \sigma^x$, what is the value of x? (A) 1/3 (B) 1/2 (C) 1 (D) 3/2 (E) 2/3

2. The helium-3 nucleus has two protons and one neutron. Which of the following statements is <u>wrong</u>? (A) The spin direction of the two protons are opposite. (B) The total spin of the nucleus is 1/2. (C) The ground state of electrons in ^{3}He atom has hyperfine structure. (D) The mass of ^{3}He nucleus is smaller than the sum of masses of two free proton and one free neutrons. (E) The neutral ^{3}He atom is a boson.

3. Which of the following statements is <u>correct</u>? (A) Photons obey Pauli exclusion principle. (B) Photon is its own antiparticle. (C) Neutron is its own anti-particle. (D) In Compton scattering, the scattered photon has wavelength shorter than that of the incident photon. (E) The strong force between a proton and a neutron is a central force and conserves angular momentum.

4. In a 3-dimensional simple harmonic potential for electrons, the eigen energy $E = (n + \frac{3}{2})\hbar\omega_0$, where $n = n_x + n_y + n_z$. n_x, n_y and n_z are 0 or positive integers. If the electrons are all in the spin-up state, and lets neglect the spin degree of freedom, how many degenerate states are there if the total energy $E = (100 + \frac{3}{2})\hbar\omega_0$? (so $n = 100$) (A) 100! (B) 102! (C) 4950 (D) 5151 (E) 5050

計算題，一題二十分。

1. (a) (10%) Using the relation for phase velocity and group velocity,

   $v_{phase} = \frac{\omega}{k}$ and $v_{group} = \frac{d\omega}{dk}$, show that the two velocities have the following relation:

   $$v_{group} = v_{phase} - \lambda \frac{d}{d\lambda}(v_{phase}) \text{ --------equation (1)}$$

   Where k is the wave number, λ is the wavelength and ω is the angular frequency of the wave.

   (b) (5%) A free particle of mass m moving with momentum $p = \hbar k$ has total energy $= \frac{\hbar^2 k^2}{2m}$,

   Find its group velocity and phase velocity in terms of p and m. Does this wave have dispersion or not?

   (c) (5%) Show that the results you obtain in (b) indeed satisfy equation (1).

2. Consider an atom with two levels of energies $E_2 > E_1$. The wave functions using Dirac bra-ket notation are $|1\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and $|2\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ respectively. The Hamiltonian of the system can then be written in matrix form as $H_0 = \begin{vmatrix}E_1 & 0\\ 0 & E_2\end{vmatrix}$. In the presence of a static electric field, the Hamiltonian is perturbed, and becomes $H = H_0 + H_I = \begin{vmatrix}E_1 & V\\ V & E_2\end{vmatrix}$, where V is proportional to the magnitude of the applied DC electric field.

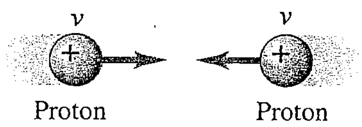
(a) (5%) Verify that $|1\rangle$ and $|2\rangle$ are the energy eigenstates of H_0, but not of H.

(b) (10%) What are the energy shifts of state 1 and state 2 due to the perturbation? Assume $E_2 - E_1 >> V$.

(c) (5%) Explain why the applied static electric field can only introduce off-diagonal terms in the Hamiltonian, but no diagonal terms.

3. In 1955, antiproton was discovered at the Bevatron accelerator at Berkeley through the following process: $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{p} + \mathrm{p} + \bar{\mathrm{p}}$. Proton has rest mass of 938 MeV/c^2.

(a) (10%) In the C.M. frame as shown in the right figure, what is the minimum velocity v of the incident protons relative to their center of mass so that the anti-proton can be produced? (You need to use relativistic energy formula.)



(b) (5%) In the real experiment, one proton is at rest while the other one is accelerating and collidues with the one at rest. What is the velocity of the moving proton as measured in the lab frame? (Lab frame is the frame of the proton at rest.)

(c) (5%) What is the minimum kinetic energy of the accelerating proton so that antiproton can be produced?

4. (5% each) The wavefunction of the 1S electron of hydrogen atom is $\psi_{1s} = \frac{1}{\sqrt{\pi}}(a_0)^{-\frac{3}{2}} \cdot e^{-\frac{r}{a_0}}$,

(a) Show that the wavefunction is properly normalized.

(b) Where is the electron most likely found? (r =?)

(c) What is the probability that the 1S electron is inside the proton? Assuming the proton is a uniform sphere with radius of 1 fm (fm = 10^{-15}m).

(d) If the electron of the hydrogen atom is replaced by a muon, the atom is called a muonic hydrogen. What is the probability that the muon is inside the proton?

In (c) and (d), we assume the finite size of the proton is a small perturbation to the Coulomb field and does not change the wavefunction of the electron or muon.

Bohr radius $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 5.29\times10^{-11}$ m; $m_e = 9.11 \times 10^{-31}$ kg; $m_\mu = 1.88 \times 10^{-28}$ kg

useful formula: $\int x^2 e^{-x} dx = -e^{-x}(x^2 + 2x + 2) + C$