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※請在答案卷內作答

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Some constants that may be useful,

Planck's constant $h=6.63\times10^{-34}$ Joule sec

$hc=(6.63\times10^{-34}$ Joule sec$)(3\times10^{8}$m/sec) ~1240 eV nm

Boltzmann constant $k_B=1.38\times10^{-23}$ Joule/K

Bohr magneton the constant is $\mu_B=5.8\times10^{-5}$ eV/T

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一、 申論題 (30 %)

(一) Why are photo-lithography room lit with yellow light? (10 %)

(二) Why the α particle can escape from the nucleus in the α decay process? (10 %)

(三) Why is the sky blue in the daytime? Why is the sky red at sunset? (10 %)

二、 計算題 (70 %)

(四) The wavefunction for a particle of mass m moving in a certain potential U(x) is given by $\Psi(x,t)=\begin{cases}(Ax)e^{-(x/L)}e^{-iEt/\hbar}, & x>0\\ 0, & x\le 0\end{cases}$ where A, L, and E are real constants. Ψ(x,t) is a properly normalized wavefunction that obeys the Schrödinger equation for the potential U(x).

1. Find the potential U(x) and the total energy E in terms of L, m, and $\hbar$. (10 %)
2. Find the spatial probability density p(x) for the particle of the total energy E. (4 %)
3. Sketch this spatial distribution p(x) and indicate the position at which the particle is most likely to be found. (6 %)
4. Determine the value of A. (5 %)

Note: $\int_{-\infty}^{\infty}\exp^{-\alpha x^2}\,dx=\sqrt{\frac{\pi}{\alpha}}$, $\int_0^{\infty}x^2e^{-\alpha x^2}dx=\frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}}$, and $\int_0^{\infty}x^2\exp^{-2\alpha x}\,dx=\frac{1}{4\alpha^3}$.

(五) A particle of mass m moves in a two dimensional box with edge lengths a and b where $U(x,y)=\begin{cases}0 & \text{for } -a<x<a \text{ and } 0<y<b\\ \infty & \text{elsewhere}\end{cases}$.

1. Find the total wave function Ψ(x,y, t) with the properly normalized factor. (10 %)
2. For the ground state, calculate the probability of finding the particle in the region where $-\frac{a}{4}\le x\le\frac{a}{4}$. (6 %)
3. Let b=2a. Find the six lowest energies in terms of a and corresponding quantum states. Which of these states are degenerate? (9 %)

Note: $\sin^2\theta=\frac{1-\cos2\theta}{2}$ and $\cos^2\theta=\frac{1+\cos2\theta}{2}$

注意：背面有試題

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（六）A cadmium (Cd) light is placed in a strong magnetic field B such that the orbital and spin magnetic dipole moments of the Cd atom precess independently about the magnetic field.

1. Derive the expression for energy shifts in terms of its quantum numbers m_ℓ and m_s. (5 %)
2. Draw the energy diagram for two excited states 5^1D_2 and 5^1P_1 in the presence of magnetic field. (8 %)
3. Assuming that red light of wavelength λ=620nm is emitted for the transition from 5^1P_1 to 5^1D_2 in the absence of magnetic field, find the energies of the emitted lights when it is placed in a field of 2 Tesla. (5 %)
4. In order to resolve these emitted lines in part 3, what accuracy is required in wavelength for the spectrometer? (2 %)