類組：物理類　科目：應用數學(2001)　

※請在答案卷內作答

1. A set of three vectors in Cartesian coordinates are $\overrightarrow{u_1}=(1,1,-1),\ \overrightarrow{u_2}=(-1,1,1),\ \overrightarrow{u_3}=(1,-1,1)$. Find a set of three vectors $\overrightarrow{v_1},\ \overrightarrow{v_2}$ and $\overrightarrow{v_3}$, such that their inner products satisfying $\overrightarrow{u_i}\bullet\overrightarrow{v_j}=\delta_{i,j}$. (10%)

2. (a) Expand exp [tan x] up to $o(x^4)$ for |x| < 1. (5%)
   (b) Expand $\tan^{-1} x$ up to $o(x^7)$ for |x| < 1. (5%)

3. (a) Find $\frac{dI}{dx}$ where $I=\int_{x^2}^{\sin^{-1}x}\frac{\sin t}{t}dt$. (5%)
   (b) Find the equation of tangent line to the curve $x^3-3y^3+xy+21=0$ at $(x,y)=(1,1)$. (5%)

4. A constant density solid ellipsoid inside the surface of $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{25}=1$,calculate the moment of inertia in terms of total mass for the rotation about the z-axis. (10%)

5. Find the solution of the ordinary differential equation
$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=4\sin(2x).$$ (10%)

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6. Let us consider a functional defined by the integral,

$$I[x]=\int_{t_0}^{t_1} L\big(x(t),\dot{x}(t),t\big)dt,$$

where the values t_0, t_1, $x(t_0)$, $x(t_1)$ are given. Here dot means the derivative with respect to t. Prove that the function $x(t)$ that minimizing/maximizing the functional $I[x]$ should satisfy

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0.$$

(a) From the condition $\delta I=0$, obtain the above equation. (5%)

(b) If the functional is defined by

$$I[x]=\int_{t_0}^{t_1} L\big(x(t),\dot{x}(t),\ddot{x}(t),t\big)dt,$$

where the values t_0, t_1, $x(t_0)$, $x(t_1)$, $\dot{x}(t_0)$, and $\dot{x}(t_1)$ are given. Obtain the differential equation that is satisfied by the function maximizing/minimizing the functional (10%).

7. Hermite polynomial is defined by the Rodrigues' formula,

$$H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}e^{-x^2}.$$

(a) Using the Goursat's theorem,

$$\frac{d^n}{dx^n}f(x)=\frac{n!}{2\pi i}\int_C\frac{f(z)}{(z-x)^{n+1}}dz,$$

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obtain the generating function $J(x,s)$ defined by

$$J(x,s)=\sum_{n=0}^{\infty}\frac{s^n}{n!}H_n(x).\ (5\%)$$

(b) Evaluate the integral, $\int_{-\infty}^{\infty}H_n(x)H_m(x)e^{-x^2}dx$. (10%)

8. Obtain the Fourier transform of the following functions: (5% each)

(a) $f(x)=\begin{cases}1 & 0\le |x|<1\\ 0 & 1\le |x|\end{cases}$　(b) $f(x)=\begin{cases}x & 0\le |x|<1\\ 0 & 1\le |x|\end{cases}$

(c) $f(x)=\begin{cases}x^2 & 0\le |x|<1\\ 0 & 1\le |x|\end{cases}$　(d) $f(x)=e^{-x^2}$