## 台灣聯合大學系統 104 學年度碩士班招生考試試題 共2 頁 第 1 頁

類組: 電機類 科目: 工程數學 A(3003)

※請在答案卷內作答

Note: Detailed derivations are required to obtain a full score for each problem.

**1.** (15 pts) Let 
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & a \end{pmatrix}$$
,  $x \in M_{3\times 1}(\mathbb{R})$ , and  $b = \begin{pmatrix} 100 \\ 200 \\ c \end{pmatrix}$ .

- (a) (5%) Find the conditions for a such that the system of equations Ax = b has a unique solution.
- (b) (5%) Find the conditions for a and c such that Ax = b has infinitely many solutions.
- (c) (5%) If Ax = b has infinitely many solutions, is it possible to find a positive integer n such that  $A^nx = b$  has a unique solution? Why or why not?
- 2. (10 pts) Let V be the vector space spanned by the set of functions  $\{1, \cos \omega t, \sin \omega t\}$ , defined on the time domain  $t \in \mathbb{R}$ . Assume that the angular frequency  $\omega \geq 0$ , and let  $\beta = \{1, \cos \omega t, \sin \omega t\}$  be regarded as an ordered basis for V. Define a linear transformation  $T: V \to V$  as follows,

$$T(x(t)) = m\frac{d^2x(t)}{dt^2} + r\frac{dx(t)}{dt} + kx(t),$$

where parameters m, r, k are non-negative.

- (a) (5%) Find the matrix representation  $A = [T]_{\beta}$ .
- (b) (5%) If  $r = 0, m \neq 0$ , find the condition for  $\omega$  such that dim (N(T)) > 0.
- 3. (9 pts) Let V be an inner product space and let T be a linear operator on V. Prove or disprove the following statement.

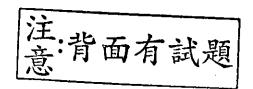
$$R(T^*)^{\perp} = N(T).$$

4. (16 pts) Given the Schur decomposition theorem as follows.

**Theorem 1** Let T be a linear operator on a finite-dimensional inner product space V. If  $det(T-tI_V)$  splits, then there exists an orthonormal basis  $\beta$  for V such that  $[T]_{\beta}$  is upper triangular.

Use this theorem to prove or disprove the following statements.

(a) (8%) Let T be a normal operator on a finite-dimensional inner product space V over  $\mathbb{C}$ . Then there exists an orthonormal basis  $\beta$  for V such that  $[T]_{\beta}$  is diagonal.



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- (b) (8%) Let T be a self-adjoint operator on a finite-dimensional inner product space V over  $\mathbb{R}$ . Then there exists an orthonormal basis  $\beta$  for V such that  $[T]_{\beta}$  is diagonal.
- 5. (20 pts) Based on the descriptions below:
  - 1. Two water tanks, denoted as  $T_1$  and  $T_2$ , are mutually connected through two pipelines.
  - 2. Initially tanks  $T_1$  and  $T_2$  contain 100 liters of water each.
  - 3. In tank  $T_1$  water is pure; while 150 grams of salt are dissolved in tank  $T_2$ .
  - 4. By stirring to keep the mixture uniform and circulating liquid through two pipelines at a rate of 2 liters per minute, the amounts of salt  $y_1(t)$  in  $T_1$  and  $y_2(t)$  in  $T_2$  change with time t.

Write down the differential equations for these two mixing tanks, and How long the tank  $T_1$  will contain at least half as much salt as there will be left in tank  $T_2$ ?

6. (10 pts) Construct a Fourier series over the interval of  $(-\pi, \pi)$  and use Parseval identity to calculate the summation of an infinite series:

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^4}.$$

- 7. (20 pts) Complex Variables:
  - (a) (5%) Prove or disapprove that  $e^{iz}$  is an entire function.
  - (b) (5%) Is  $Re[\oint f(z) dz] = \oint Re[f(z)] dz$ ? Explain.
  - (c) (5%) Given that  $Ln(1+z) = z \frac{z^2}{2} + \frac{z^3}{3} + \cdots$  for |z| < 1, find the first three terms of the Taylor series of Ln(2z) with the center at -2i. Also find its radius of convergence.
  - (d) (5%) Evaluate

$$\oint_C \frac{\exp(-z)}{\cos(z)} \, dz,$$

C: |z| = 2, counter clockwise.

注:背面有試題