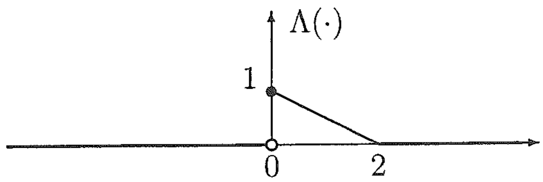
計算題（計算題應詳列計算過程，無計算過程者不予計分）

1. For the graph below, answer the following questions.



Note that the argument of $\Lambda(\cdot)$ will be specified in the subproblems.

(a) (3%) Can $\Lambda(\tau)$ be the autocorrelation function of a (possibly complex-valued) wide-sense stationary (WSS) random process? Justify your answer.

(b) (3%) Can $\Lambda(f)$ be the power spectrum density (PSD) of a real-valued WSS random process? Justify your answer.

(c) (3%) Suppose $\lambda(t)$ is the inverse Fourier transform of $\Lambda(f)$. Plot the Fourier transform of $\lambda(2t)$.

(d) (3%) Suppose $a(t) \triangleq \sum_{n=-\infty}^{\infty} \lambda(t-n)$, where $\lambda(t)$ is the inverse Fourier transform of $\Lambda(f)$. Plot the Fourier transform of $a(t)$.

(e) (4%) Plot the Fourier transform of $s(t) \triangleq \mathrm{Re}\{\lambda(t)e^{j2\pi f_c t}\}$, where $\lambda(t)$ is the inverse Fourier transform of $\Lambda(f)$ and $f_c \gg 2$.

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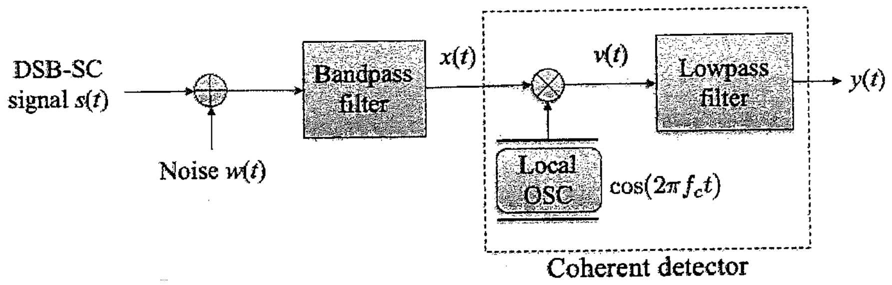
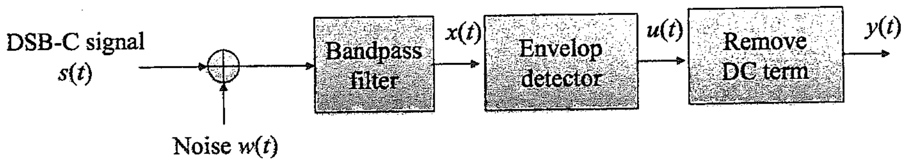
2. Below are the diagrams of the DSB-SC coherent receiver and the DSB-C envelop detector, respectively.





In both diagrams, $w(t)$ is the additive white Gaussian noise with two-sided PSD $N_0/2$, and

$$x(t) = s(t) + n(t),$$

where

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

is the filtered white noise with $\mathbb{E}[n(t)] = \mathbb{E}[n_I(t)] = \mathbb{E}[n_Q(t)] = 0$ and $\mathbb{E}[n^2(t)] = \mathbb{E}[n_I^2(t)] = \mathbb{E}[n_Q^2(t)] = 2WN_0$. Let $m(t)$ be a zero-mean wide-sense stationary (WSS) process with $\mathbb{E}[m^2(t)] = P$ and with bandwidth W.

(a) (7%) Find the output signal-to-noise ratio SNR_O of the DSB-SC coherent receiver, where $s(t) = A_c m(t)\cos(2\pi f_c t)$. Note that $y(t)$ is the output induced by passing input $v(t)$ via an ideal lowpass filter of bandwidth W.

Hint: SNR_O is the average power of the *signal content* in $y(t)$ divided by the average power of the *noise content* in $y(t)$.

(b) (7%) Subject to

$$\left|A_c[1 + k_a m(t)]\right| \gg \sqrt{n_I^2(t) + n_Q^2(t)},$$

find the SNR_O of the DSB-C envelop detector, where

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t).$$

Note that the output of the envelop detector due to input $x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$ is given by $u(t) = \sqrt{x_I^2(t) + x_Q^2(t)}$, and $y(t) = u(t) - \mathbb{E}[u(t)]$.

3. Let $x(t) \triangleq a \cdot g(t) + w(t)$, $0 \leq t \leq T$, where $a = 1$ or -1, $g(t)$ is a nonzero deterministic pulse signal, and $w(t)$ is a white noise process of zero mean and power spectral density $\frac{N_0}{2}$. Let $h(t)$ be the impulse response of a linear filter, and let $y(t) \triangleq x(t) \star h(t) = g_o(t) + w_o(t)$, where $\star$ denotes convolution, $g_o(t) \triangleq a \cdot g(t) \star h(t)$, and $w_o(t) \triangleq w(t) \star h(t)$. We wish to find $h(t)$ such that the signal-to-noise ratio $|g_o(T)|^2 / E[w_o^2(T)]$ at $t = T$ is maximized.

   (a) (3%) For $g(t) = \frac{T-t}{T}$, $0 \leq t \leq T$, what is the desirable $h(t)$?

   (b) (12%) For given arbitrary $g(t)$, write the desirable $h(t)$ in terms of $g(t)$ and show how you obtain the $h(t)$. (Hint: you may need Schwarz's inequality:

$$\left| \int_{-\infty}^{\infty} \phi_1(t)\phi_2(t)\, dt \right|^2 \leq \int_{-\infty}^{\infty} |\phi_1(t)|^2\, dt \int_{-\infty}^{\infty} |\phi_2(t)|^2\, dt$$

   with equality if and only if $\phi_1(t) = \gamma \phi_2^*(t)$, where γ is any nonzero number and $*$ denotes complex conjugate.)

   (c) (7%) Assume that $a = 1$ or $a = -1$ with equal probability, and we obtain a normalized quantity $\tilde{y}(T) = a + \tilde{w}$, where $\tilde{w}$ is Gaussian r.v. with zero mean and variance σ^2. Form a maximum-likelihood decision $\tilde{a}$ of a given $\tilde{y}(T)$, and derive the probability $\Pr(\tilde{a} \neq a)$. (Hint: The pdf of a zero-mean Gaussian r.v. ν with variance σ^2 is

$$f(\nu) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\nu^2}{2\sigma^2} \right).$$

   Please express the probability $\Pr(\tilde{a} \neq a)$ in terms of

$$\mathrm{erfc}(u) \triangleq \frac{2}{\sqrt{\pi}} \int_u^{\infty} \exp(-z^2)\, dz.$$

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4. Let $p(t)$ be a continuous time function, and let $P(f)$ be the Fourier transform of $p(t)$, defined by

$$P(f) \triangleq \begin{cases} \frac{W-|f|}{W}, & -W \leq f \leq W \\ 0, & f \notin [-W, W] \end{cases}$$

where W is some constant with $W > 1$.

   (a) (4%) Let $T = \frac{1}{W}$. What is the discrete time function $a[n] \triangleq p(nT)$ for $n = 0, \pm 1, \pm 2, \ldots$?

   (b) (4%) Let $H(f)$ be the frequency response of a time function $h(t)$ with

$$H(f) \triangleq \begin{cases} 1, & -W \leq f \leq W \\ \frac{2W-|f|}{W}, & W \leq |f| \leq 2W \\ 0, & f \notin [-2W, 2W]. \end{cases}$$

   Let $p[n] \triangleq p(nT)$ for $n = 0, \pm 1, \pm 2, \ldots$, where T is a positive number, not yet specified. Can you reconstruct $p(t)$ from $p[n]$ with $h(t)$? If yes, how do you make it? (Hint: Specify the value of T yourself.)

5. (10%) Consider a direct sequence spread spectrum system with an m-sequence $\mathbf{c}$ of length n and message $u \in \{-1, +1\}$ with probability $\Pr(u = -1) = 0.8$ and the input signal is $\mathbf{x} = \sqrt{E_b} u \mathbf{c}$. The received signal is

$$\mathbf{y} = \mathbf{x} + \mathbf{w}, \quad \text{where } \mathbf{w} \sim \mathcal{N}(\mathbf{0}, (N_0/2)\mathbf{I}).$$

Show that the *optimal* receiver is of the form $\mathbf{c}^T\mathbf{y} \underset{-1}{\overset{+1}{\gtrless}} \eta$. Derive the η. Also, derive the processing gain.

6. Let X be a random variable with the probability mass function [0.48, 0.26, 0.13, 0.03, 0.02, 0.04, 0.04].

   (a) (7%) Construct two different Huffman codes whose sets of codeword lengths are not the same.

   (b) (7%) Compute the expected lengths of the two codes constructed in part (a). Also, are the expected lengths smaller than the entropy $H(X)$ (in bits)?

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7. Let $\mathcal{C}$ be a binary linear code with the following parity check matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

(a) (6%) Find a generator matrix $\mathbf{G}$ for $\mathcal{C}$.

(b) (4%) Determine the code rate of $\mathcal{C}$.

(c) (6%) Let $\mathbf{v} = [1, 0, 1, 1, 0, 0, 0]$ be the received signal. Compute the syndrome. Find an error vector of weight 1 that has the same syndrome of $\mathbf{v}$. Use syndrome decoder to decode $\mathbf{v}$.