※請在答案卷內作答

考生請注意

- 本試卷分為填充題及問答題兩部分。
- 作答填充題時，請依題號順序，於答案卷作答區**第一頁**標清題號依序作答，並清楚標示答案。作答填充題時**請勿**附加推導過程。
- 作答問答題時，請依題號順序，於答案卷作答區**第二頁**起標清題號依序作答。作答問答題時**必須**寫出推導過程。
- 你也許會需要這個：Q 函數值表，$Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{u^2}{2}}du$。

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0	0.5000	1.0	0.1587	2.0	0.0228	3.0	0.0013
0.1	0.4602	1.1	0.1357	2.1	0.0179	3.1	0.0010
0.2	0.4207	1.2	0.1151	2.2	0.0139	3.2	0.0007
0.3	0.3821	1.3	0.0968	2.3	0.0107	3.3	0.0005
0.4	0.3446	1.4	0.0808	2.4	0.0082	3.4	0.0003
0.5	0.3085	1.5	0.0668	2.5	0.0062	3.5	0.0002
0.6	0.2743	1.6	0.0548	2.6	0.0047	3.6	0.0002
0.7	0.2420	1.7	0.0446	2.7	0.0035	3.7	0.0001
0.8	0.2119	1.8	0.0359	2.8	0.0026	3.8	0.0001
0.9	0.1841	1.9	0.0287	2.9	0.0019	3.9	0.0000

第一部分、填充題(請勿附加推導過程)

1. (Total=15%) Fig. 1 shows a series of operations that are used to separate random processes into different frequency regions. The input random process $x(t)$ is Gaussian and white with a power spectral density function of $S_x(f) = 2$.

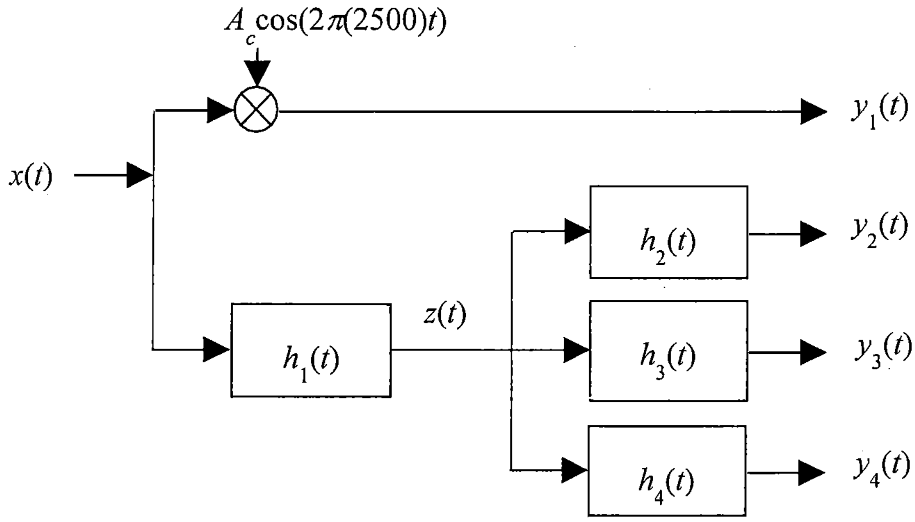

Fig 1. Problem 1.

The system functions of the various linear filters are as follows

$$H_1(f) = \begin{cases} 1, & 1000 \le |f| \le 3000 \\ 0, & \text{otherwise} \end{cases}$$

$$H_2(f) = \begin{cases} 1, & 1000 \le |f| \le 2000 \\ 0, & \text{otherwise} \end{cases}$$

$$H_3(f) = \begin{cases} 1, & 1500 \le |f| \le 2500 \\ 0, & \text{otherwise} \end{cases}$$

$$H_4(f) = \begin{cases} 1, & 2000 \le |f| \le 3000 \\ 0, & \text{otherwise} \end{cases}$$

We developed expressions that relate the various random processes that are inputs and outputs to linear filters. For example, recall that

$$r_{zz}(\tau) = r_{xx}(\tau) * h_1(\tau) * h_1(-\tau)$$
$$r_{zx}(\tau) = r_{xx}(\tau) * h_1(\tau)$$
$$r_{y2y3}(\tau) = r_{zz}(\tau) * h_2(\tau) * h_3(-\tau)$$

Use the above properties, your knowledge of Fourier transforms, and your general knowledge of the properties of random processes to answer the following questions.

(a) (4%) Considering $y_1(t)$, $y_2(t)$, $y_3(t)$, and $y_4(t)$, the random processes that are wide-sense stationary are ____________________

(b) (3%) Considering $y_1(t)$, $y_2(t)$, $y_3(t)$, and $y_4(t)$, the random processes that are white are ________________________

(c) (2%) It is _____________(true/false) that the random processes $y_2(t)$ and $y_3(t)$ are statistically independent.

(d) (6%) The probability that $z(t)$ is greater than 3 at t=20 is _____________

2. (Total=18%) A pulse communication system is being designed that will transmit one of two messages, m_1 and m_2, which occur with equal *a priori* probability. These messages will be signaled using the following waveforms, and we would like to select sets of transmitted signals that will either minimize the average transmitted energy or will minimize the probability of error. Assume that the channel is corrupted by an additive white Gaussian random process. Assume here that the inner product of two real-valued function of duration $T = 2$ is defined as

$$\langle f(t), g(t)\rangle \triangleq \int_0^T f(t)g(t)\ dt.$$

Four possible signals, $s_A(t)$, $s_B(t)$, $s_C(t)$, and $s_D(t)$ are being considered, and two of them will be selected as the signal set that will indicate which message had been sent. The signals are sketched in Fig. 2.

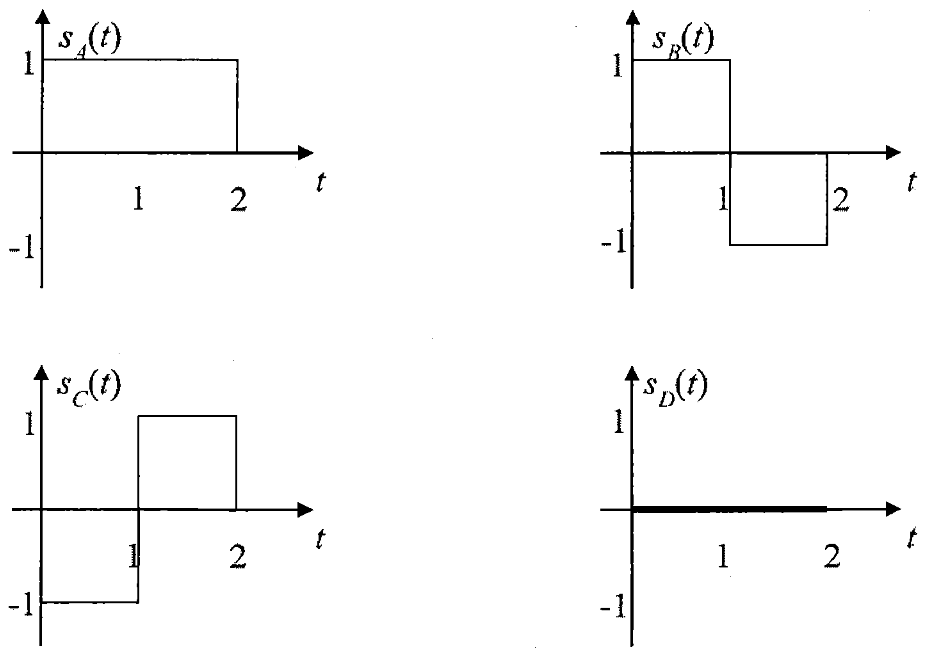

Fig 2. Problem 2.

(a) (4%) Compute a set of orthonormal basis functions $\{\phi_A(t), \phi_B(t), \phi_C(t), \phi_D(t)\}$ using the Gram-Schmidt procedure, where $\phi_A(t)$ is proportional to $s_A(t)$. The signals $s_A(t)$, $s_B(t)$, $s_C(t)$, and $s_D(t)$ can be expressed as a linear combination of these basis functions as

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where the basis functions are

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(b) (4%) The TWO candidate signals that would result in minimum average transmitted energy, given that the two messages are equiprobable, are ________________________.

(c) (4%) The TWO candidate signals that would provide the lowest average probability of a bit error are ________________________

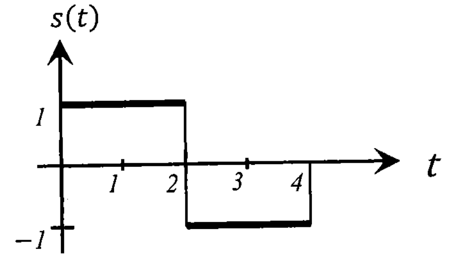
(d) (6%) To increase data rate, suppose now you want to send THREE messages instead of two, which again occur with equal *a priori* probability. Suppose $s_A(t)$, $s_B(t)$, and $s_C(t)$ are chosen to represent these messages, and the power spectral density of the noise is $\frac{N_0}{2} = 1$. The union bound of the average symbol error probability is ________________________

第二部分、問答題(請寫出推導過程)

3. (Total=15%) Consider a communication system in which the received signal $y(t)$ is given by

$$y(t) = \begin{cases} s(t) + w(t), & \text{if "1" is sent,} \\ w(t), & \text{if "0" is sent,} \end{cases}$$

where $w(t)$ is white Gaussian noise with (zero mean) power spectral density $N_0/2$ and $s(t)$ is shown below



At the receiver, the received signal $y(t)$ is passed through a filter with impulse response $h(t) = s(T - t)$. The output of the filter is sampled at time t_0 that provides a decision statistic Y with

$$Y = \begin{cases} S + W, & \text{if "1" is sent,} \\ W, & \text{if "0" is sent,} \end{cases}$$

where $S = s(t) * h(t)|_{t=t_0}$ and $W = w(t) * h(t)|_{t=t_0}$ are the filtered/sampled outputs of $s(t)$ and $w(t)$, respectively. The maximum likelihood (ML) decision rule is then used based on Y to decide whether "1" or "0" is sent. Assume the noise process is independent of which signal is sent.

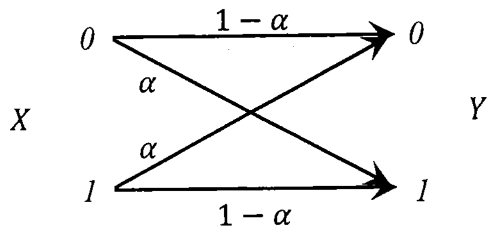
(a) (3%) Let $t_0 = 3$ and T=4, i.e. $h(t) = s(4 - t)$. Determine the mean and variance of the filtered and sampled noise term W.

(b) (6%) Again, let $t_0 = 3$ and T=4. Please find the corresponding bit error rate (BER) of the ML receiver. (Express your answer in Q-function.)

(c) (3%) Can the BER in part (b) with $h(t) = s(4 - t)$ be improved by choosing a different sampling time t_0? Justify your answers.

(d) (3%) Suppose now T=5, i.e. $h(t) = s(5 - t)$. How to determine the sampling instant t_0 in this case? Please provide a general rule to determine the relation between the sampling instant t_0 and the delay T such that the sampled output yields the best BER.

4. (Total=18%) Consider a binary symmetric channel with input X, output Y, and transition probability α. More specifically, as shown in the figure below, the input and output of the channel may be "0" or "1", $P(Y=0|X=0)=P(Y=1|X=1)=1-\alpha$ and $P(Y=1|X=0)=P(Y=0|X=1)=\alpha$. The prior probability is $P(X=0)=p$.



(a) (4%) Find the entropy $H(Y)$ and the conditional entropy $H(Y|X)$.

(b) (4%) Find α that can maximize the channel capacity.

(c) (5%) For the case of $p<\alpha$, please give the maximum *a posteriori* probability (MAP) decision rule to decide which X is sent based on the output Y. (Hint: In this problem, the output Y of the channel can take just two values "0" or "1". Hence, it is sufficient to find the decision result for these two output possibilities.)

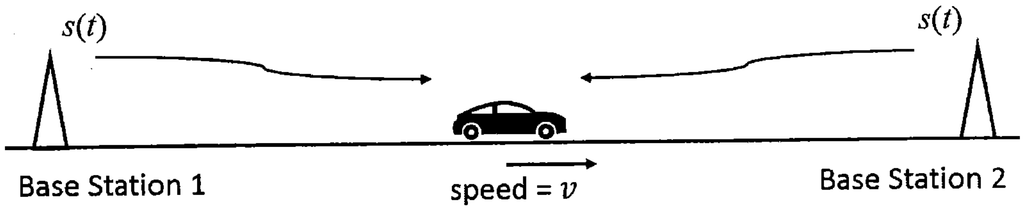
(d) (5%) Suppose the prior probability is not known in advance. We manage to produce an *estimated* prior probability $\hat{P}(X=0)=q$, which may or may not be equal to the *true* prior probability $P(X=0)=p$. In this case, the *cross-entropy* between the true and estimated prior distributions P and $\hat{P}$ is defined by $H(P,\hat{P}) \triangleq -p\log_2 q-(1-p)\log_2(1-q)$, which can be considered as an approximated entropy of X. Please show that the cross-entropy $H(P,\hat{P})$ is always no less than the true entropy $H(X)$ of X, i.e. $H(P,\hat{P}) \geq H(X)$.

(Hint: You may use the Jensen's inequality: $p\log_2 a+(1-p)\log_2 b \leq \log_2(pa+(1-p)b)$ for $0 \leq p \leq 1$, $a>0$, and $b>0$.)

5. (Total=34%) The figure below depicts a scenario where two base stations transmit simultaneously the same Quadrature Amplitude Modulation (QAM) signal to a car that is in the middle of two base stations and moves toward Base Station 2. The signal is expressed as

$$s(t) = A_c p(t)\cos(2\pi f_0 t) - A_s p(t)\sin(2\pi f_0 t)$$

in which f_0 is the carrier frequency, $p(t)$ is a rectangular pulse with the width of T, and the amplitudes A_c and A_s can take on values from $\{\pm 1, \pm 3\}$.



(a) (5%) Sketch the signal space diagram (constellation diagram) of this QAM signal.

(b) (10%) Sketch the block diagram of the coherent receiver of a typical QAM system. Furthermore, derive what happens if the local oscillation used by the "coherent" receiver is incorrect (say, the frequency is $f_0 + \Delta f$).

(c) (5%) Let the symbol error rate of QAM be approximated as $4Q(\sqrt{\gamma})$ where Q represents the Q function and γ is the SNR. To improve the symbol error rate from 10^{-2} to 10^{-3}, roughly how much shall the SNR increase?

(d) (7%) Assume that the channel gain and propagation delay from two base stations to the car are the same. Derive the mathematical expression of the received signal, ignoring the speed of the car.

(e) (7%) Now consider the car's speed v and the Doppler effect (the carrier frequency is modified with $f = \frac{c \pm v}{c} f_0$, where c is the speed of light, and choosing the sign according to the moving direction). Derive the mathematical expression of the received signal.