台灣聯合大學系統109學年度碩士班招生考試試題

類組：電機類　科目：通訊系統(通訊原理)(300E)　　共 3 頁 第 1 頁

計算題請寫出過程。

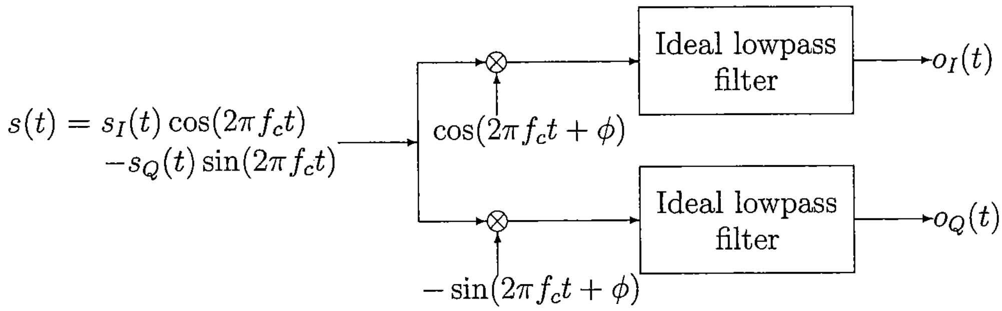
1. (a) (4%) Explain why $R_X(\tau) = \sin(2\pi f_c \tau)$ cannot be the autocorrelation function of a wide-sense stationary (WSS) random process $X(t)$.

   (b) (6%) Below is a list of the analog modulation schemes for the transmission of message $m(t)$, where the transmission signal is modeled as $s(t) = \mathrm{Re}\left\{(s_I(t) + j s_Q(t))e^{j2\pi f_c t}\right\}$. Provide what should be placed in the three blanks under $s_Q(t)$ column below.

Modulations	$s_I(t)$	$s_Q(t)$	
DSB-SC	$m(t)$	0	
SSB	$m(t)$	((b1))	Upper sideband transmission
SSB	$m(t)$	((b2))	Lower sideband transmission
DSC-C (AM)	$[1+k_a m(t)]$	((b3))	

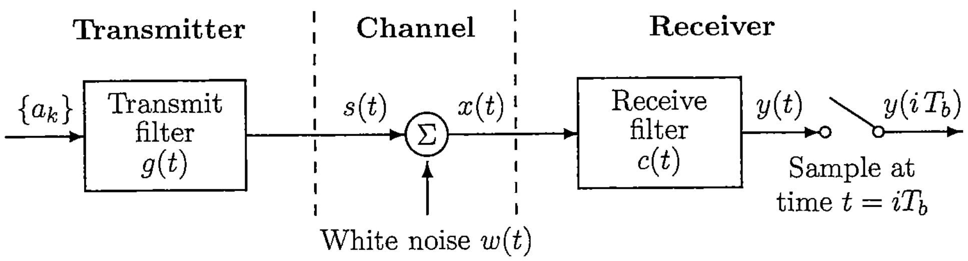
   Note: DSB-C = Double-sideband with carrier, DSB-SC = Double-sideband with suppressed carrier, SSB = Single-sideband

   (c) (6%) Suppose the oscillator at the receiver has a phase difference ϕ to the oscillator at the transmitter, and suppose the value of ϕ can be perfectly estimated via a separate low-power pilot tone. Show that we can demodulate $s_I(t)$ and remove $s_Q(t)$ from $s(t)$ for all the above four modulations by two product modulators and two lowpass filters as shown below.



   Note: You shall represent $s_I(t)$ as a function of $o_I(t)$, $o_Q(t)$ and ϕ.

2.



In the above diagram, we obtain that for sequence transmission $\{a_k\}$,

$$y(iT_b) = \sum_{k=-\infty}^{\infty} a_k \cdot p((i-k)T_b) + \underbrace{w(t) \star c(t)\big|_{t=iT_b}}_{=n_i},$$

where $p(t) = g(t) \star c(t)$, and "$\star$" is the convolution operation. Denote by $G(f)$, $C(f)$ and $P(f)$ the Fourier transforms of $g(t)$, $c(t)$ and $p(t)$, respectively. The noise $w(t)$ is zero-mean and white with power spectra density $\frac{N_0}{2}$.

注意:背面有試題

(a) (6%) If $n_i = 0$ and the function $p(t)$ satisfies

$$p(kT_b) = \begin{cases} 1, & k = k_0; \\ 0, & k \neq k_0, \end{cases} \tag{1}$$

determine $y(iT_b)$.

(b) (6%) Determine $\sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_b}\right)$ based on (1).
Hint: $P_\delta(f) = \frac{1}{T_b}\sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_b}\right)$ is the Fourier transform of $p_\delta(t) = \sum_{n=-\infty}^{\infty} p(nT_b)\,\delta(t - nT_b)$.

(c) (6%) Determine the filter $c(t)$ that maximizes the output signal-to-noise ratio (SNR) for given $g(t)$. You may suppose a_0 is transmitted, and examine the SNR of $y(iT_b)|_{i=0} = y(T_b)$.

3. Let $\phi_1(t), \ldots, \phi_N(t)$, $0 \le t \le T$, be real-valued orthonormal functions, and let $s_i(t) \triangleq \sum_{j=1}^{N} s_{ij}\phi_j(t)$ for $i = 1, 2, \ldots, M$ be real-valued signals. Define $\mathbf{s}_i \triangleq (s_{i1}, s_{i2}, \ldots, s_{iN})^\mathsf{T}$ where T denotes the transpose of the vector. (Note that $\|\mathbf{s}_i\|^2 \triangleq \mathbf{s}_i^\mathsf{T}\mathbf{s}_i$.)

(a) (6%) Show that $\int_0^T s_i(t)s_j(t)\,dt = \mathbf{s}_i^\mathsf{T}\mathbf{s}_j$ and $\int_0^T \left(s_i(t) - s_j(t)\right)^2 dt = \|\mathbf{s}_i - \mathbf{s}_j\|^2$.

Let $W(t)$ be a real-valued Gaussian process with zero-mean and the autocorrelation function $R_W(t,u) \triangleq \frac{N_0}{2}\delta(t-u)$. Define, for $j = 1, 2, \ldots, N$, the random variables

$$W_j \triangleq \int_0^T W(t)\phi_j(t)\,dt.$$

(b) (6%) Derive $E\{W_j^2\}$ and $E\{W_jW_k\}$ for $j \neq k$.

Now, assume $M = 2$, and that $s_1(t) \triangleq 1, 0 \le t \le T$ and

$$s_2(t) \triangleq \begin{cases} 1, & 0 \le t \le \frac{T}{2} \\ -1, & \frac{T}{2} \le t \le T \end{cases}$$

Moreover, assume that we observe $x(t) = s_i(t) + w(t)$, $0 \le t \le T$, with $i = 1$ or 2, and $w(t)$ a sample realization of the Gaussian process $W(t)$ defined above.

(c) (3%) Find a smallest set of orthonormal functions $\phi_1(t), \ldots, \phi_N(t)$ such that $s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t)$ for $i = 1, 2$. Let $\mathbf{s}_1$ and $\mathbf{s}_2$ be vectors as defined above. Compute $d \triangleq \|\mathbf{s}_1 - \mathbf{s}_2\|$.

(d) (6%) Give a maximum likelihood (ML) decision rule for estimate $\tilde{s}(t)$ of the signal $s_i(t)$ in $x(t)$, and derive the error probability of the ML estimate $\tilde{s}(t)$, i.e., compute $P\left(\tilde{s}(t) \neq s_i(t)\right)$.
Please express $P\left(\tilde{s}(t) \neq s_i(t)\right)$ in terms of the function

$$\operatorname{erfc}(u) \triangleq \frac{2}{\sqrt{\pi}}\int_u^\infty \exp(-z^2)\,dz.$$

You may need the probability density function (pdf) $f(\nu)$ of a Gaussian r.v. ν with zero mean and variance $\frac{N_0}{2}$:

$$f(\nu) = \frac{1}{\sqrt{\pi N_0}}\exp\left(-\frac{\nu^2}{N_0}\right)$$

4. (This problem is a continuation of Problem 3, but can be solved independently.) Now, assume $M = 4$ and consider the passband signals

$$s_i(t) \triangleq a_i \cdot \sqrt{\frac{2E}{T}} \cos(2\pi f_c t), \qquad 0 \le t \le T$$

for $i = 1, 2, 3, 4$ with $a_1 = 1, a_2 = 3, a_3 = 5, a_4 = 7$, and f_c an integer multiple of $\frac{1}{T}$.

 (a) (1%) Define orthonormal function $\phi_1(t), 0 \le t \le T$, and specify with respect to $\phi_1(t)$ the message points $\mathbf{s}_i$ for $i = 1, 2, 3, 4$.

 (b) (3%) Draw a constellation of the message points $\mathbf{s}_i$, and label each $\mathbf{s}_i$ with 2 bits using Gray mapping. Give the value of $d_{\min} \triangleq \min\{\|\mathbf{s}_i - \mathbf{s}_j\| : i \ne j\}$.

 (c) (8%) Suppose that $x(t) = s_i(t) + w(t)$ for some $s_i(t)$ with probability $P(s_i(t)) = 1/4$ for each i, and $w(t)$ a sample realization of $W(t)$ as defined in Problem 3. Form a ML estimate $\tilde{s}(t)$ of the signal $s_i(t)$, given $x(t)$; derive the averaged probability of *symbol* error $P\big(\tilde{s}(t) \ne s_i(t)\big)$.
 Hint: you may need erfc(.) and the pdf $f(.)$ given in Problem 3(g).

5. Consider binary data modulation at a carrier frequency of f_c in a Rayleigh fading channel, given that the complex envelope of the received signal $y(t)$ be written as $\tilde{y}(t) = A\, e^{j\Phi}\, d(t)$, if the AWGN noise term (with power spectral density $S_N(f) = N_0/2$) is not included, where $d(t)$ is a NRZ waveform with an amplitude of 1 and with bit period T, A is a random amplitude with Rayleigh pdf $f_A(a) = (a/\sigma^2)\exp(-a^2/(2\sigma^2))$, and Φ is random phase uniformly distributed between 0 and 2π.

 (a) (4%) Find out the equation of the received signal $y(t)$ and explain the effect of the channel on the received signal.

 (b) (4%) Assume DPSK is used and given that the bit error probability of DPSK in AWGN as $P_e = (1/2)e^{-E_b/N_0}$, prove that DPSK over a Rayleigh fading channel has an average bit error rate of $1/[2(1+\gamma_0)]$, where γ_0 is the average E_b/N_0.

 (c) (4%) If BPSK modulation is used instead, draw the block diagram of an L antenna space diversity receiver structure with a maximal ratio combiner and explain how it works.

 (d) (4%) Following (c), prove that the maximal ratio combiner provides the largest SNR for data detection among all linear combining schemes.

6. Consider a $(7, 4)$ systematic cyclic code (a codeword consists of 4 message bits on the left and 3 parity bits on the right) generated by a generator polynomial $g(x) = x^3 + x^2 + 1$.

 (a) (3%) Prove that $g(x)$ is a primitive polynomial.

 (b) (4%) Explain how the message bits 0111 is encoded into a systematic codeword.

 (c) (5%) For a received codeword 0111011, explain how the syndrome is calculated. Explain why the syndrome can be used for error detection and comment on the reliability of this syndrome-based error detection method.

 (d) (5%) Explain why the syndrome can also be used for error correction when there is only a single bit error in the received codeword.