※請在答案卷內作答

參考用

一、問答、計算題（共 9 題：合計 100 分）

答題說明：

1. 請依題號順序書寫於答案卷，並清楚標註題號。

2. 每題題目前會說明配分。例如：[5 points]即代表本子題答對可得五分。

1. [10 points] To encourage collaborative study, the TA is considering assigning each student to a study group with two or three other students. Prove that as long as the enrollment is large enough, the class can always be divided into such study groups.

2. [6 pts] Explain, without using a truth table, why $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p, q, and r have the same truth value and it is false otherwise.

3. [8 pts] Please prove or disprove that the 5-clique K_5 is a planar graph.

4. [20 pts:4 each] For each row in the following table, determine whether there exist functions f and g that satisfy all the properties marked "Yes" and do not satisfy the properties marked "No". If the answer is "Yes", please provide an example of f and g.

	$f = \theta(g)$	$f = O(g)$	$f = o(g)$	$f = \Omega(g)$	$f = \omega(g)$	Do f, g exist?	Example
(a)	Yes	Yes	Yes	No	No		
(b)	No	No	No	Yes	Yes		
(c)	No	No	Yes	No	No		
(d)	Yes	Yes	No	Yes	No		
(e)	No	No	No	No	No		

注意：背面有試題

5. [10 pts] Solve the following problem using the Pigeonhole Principle. Try to identify the pigeons, the pigeonholes, and a rule assigning each pigeon to a pigeonhole. Every ID number of students in UCCU starts with a 9. Suppose that each of the 102 students in discrete math class sums the nine digits of his or her ID number and doubles the result. Explain why two students' results must be the same.

6. [9 points] Classify each of the binary relations as one of the following relations and briefly explain why.
E: An equivalence relation.
T: A Total order.
P: A Partial order that is not total.
S: A Symmetric relation that is not transitive.
N: None of the above.

(a) [3 points] The relation xRy between times of day such that x and y are at most twenty minutes apart.

(b) [3 points] The relation xRy over all words in this sentence such that x does not appear after y. (Consider "x", "y", and "xRy" to be words.)

(c) [3 points] The relation xRy over all words in this sentence such that the final appearance of y occurs after x.

7. [20 points] Given two graphs G and H in the following figures (a) and (b), respectively,

(a) [5 points] Are G and H isomorphism? If yes, please provide the mapping function. If no, please justify your answer.

(b) [5 points] Please build the minimum spanning tree of graph G.

(c) [5 points] In a graph, we define the distance between two vertices to be the length of the shortest path between them. We define the diameter of a graph to be the largest distance between any two nodes. What is the diameter of graph G? Explain why.

(d) [5 points] Give a coloring of the vertices of G that uses the minimum number of colors.
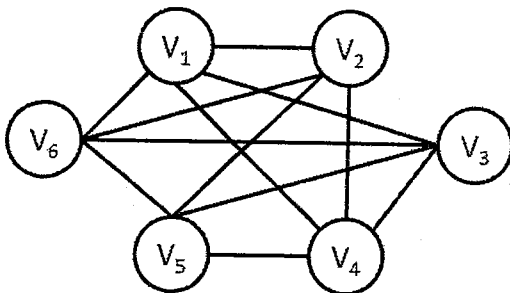
(a) Graph G
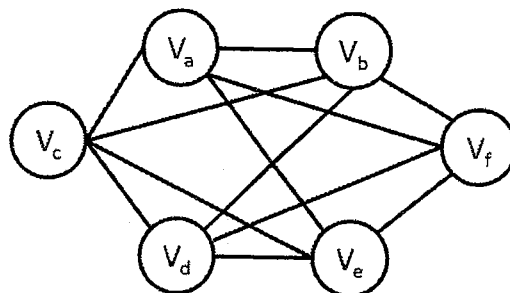
(b) Graph H

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8. [10 points] Find the multiplicative inverse of 19 modulo 79 in the range $\{0, 1, \cdots, 79\}$. (Find x, where $19x \equiv 1 \pmod{79}$)

9. [7 points] Give a combinatorial proof that $\sum_{k=1}^{n} k(C_k^n)^2 = nC_{n-1}^{2n-1}$. [Hint: Count in two ways the number of ways to select a committee, with n members from a group of n ECE professors and n computer science professors, such that the chairman of the committee is an ECE professor.]

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