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※請在答案卷內作答

一、是非題(共5題，每題4分：合計20分，答錯會倒扣4分)

答題說明：1. 請依題號順序書寫於答案卷，並清楚標註題號。

2. 詢問內容認為描述正確者書寫T，錯誤則書寫F。其餘答案一律不給分。

1. (T or F) $\neg(P \wedge (Q \vee R)) = \neg P \vee (\neg Q \vee \neg R)$

2. (T or F) At a seminar in NCTU, 113 students showed up. During the seminar, some students shook hands with each other. An old man under the bridge claimed that each student except Bob shook hands with exactly 19 other students, while Bob only shook hands with 2 other students. Is it possible? (T for Possible, and F for impossible)

3. (T or F) R is an equivalence relation in the following statement.
R:={(x,y)∈W ×W | the words x and y have at least one letter in common}, where W is the set of all words in the English dictionary.

4. (T or F) If $a \cdot 201 - m \cdot 97 = 1$, this guarantee that a has a multiplicative inverse mod m.

5. (T or F) Suppose Bob repeatedly flips a fair coin until he see the sequence HTT or HHT. The probability Bob sees the sequence HTT first is 1/2.

二、問答/計算題(共6題：合計80分)

答題說明：1. 請依題號順序書寫於答案卷，並清楚標註題號。

2. 每題題目前會說明配分。例如：[5 pts]即代表本子題五分。

6. [10 pts] Let $G_0 = 1$, $G_1 = 3$, $G_2 = 9$, and define $G_n = G_{n-1} + 3G_{n-2} + 3G_{n-3}$ for $n \geq 3$. Show by induction that $G_n \leq 3^n$ for all $n \geq 0$.

注意：背面有試題

7. [15 pts] Let S be a set of students, C be a set of courses, and E be a predicate on $S \times C$ such that $E(x, y)$ states that student x is enrolled in course y. Translate the English description of each predicate or proposition below into a logical formula using quantifiers. Note that you are **NOT** allowed to use the "set size" operation, i.e., you are not allowed to use $|A|$ for any set A. **ONLY** use $\wedge$, $\vee$, $\neg$, $\rightarrow$, $\leftrightarrow$, $\neq$, $\forall$, and $\exists$.

(a) [5 pts] Happy Potter is enrolled in all courses except for EE9527. (Here Happy Potter is an element of S and EE9527 is an element of C.)

(b) [5 pts] No matter which two courses you consider, some student is enrolled in both.

(c) [5 pts] No two students have the same set of courses.

8. [20 pts] The adjacency matrix A of a graph is given below, where each entry (i,j) represents the connection between node i and j. For example, if $A_{3,4} = 1$, node 3 and node 4 are connected with an edge. If $A_{3,4} = 0$, node 3 and node 4 are disconnected.

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(a) [5 pts] Draw the graph defined by this adjacency matrix. Label the vertices of your graph $1, 2, \cdots, 5$ so that vertex i corresponds to row and column i of the matrix.

(b) [5 pts] In a graph, we define the distance between to vertices to be the length of the shortest path between them. We define the diameter of a graph to be the largest distance between any two nodes. What is the diameter of this graph? Explain why.

(c) [5 pts] Give a coloring of the vertices that uses the minimum number of colors.

(d) [5 pts] Now we have the adjacency matrix B of a new graph G' is given below, where each entry (i,j) represents the distance between node i and j. For example, if $A_{3,4} = 4$, node 3 and node 4 are connected with an edge of distance 4. If $A_{3,4} = \infty$, node 3 and node 4 are NOT DIRECTLY connected. Draw the minimum spanning tree of G'.

$$B = \begin{bmatrix} 0 & 2 & 5 & \infty & \infty \\ 2 & 0 & 4 & 3 & \infty \\ 5 & 4 & 0 & 8 & 4 \\ \infty & 3 & 8 & 0 & 5 \\ \infty & \infty & 4 & 5 & 0 \end{bmatrix}$$

注意：背面有試題

9. **[15 pts]** Find Θ bounds for the following divide-and-conquer recurrences. Assume $T(1) = 1$ in all cases. Show your work.
(a) [7 pts] $T(n) = 2T(\lceil n/8 \rceil + 1/n) + n$
(b) [8 pts] $T(n) = 7T(\lceil n/20 \rceil) + 2T(\lceil n/8 \rceil) + n$

10. **[10 pts]** One country only has two denominations of paper currency, i.e., one is 3^9 and another one is 5^7. Suppose two persons ("A" and "B") in this country has infinite money, i.e., infinite 3^9-dollar and 5^7-dollar bills. Can "A" make a payment of 1 dollar to B? Please briefly explain why or why not. For example, "A" can pay "B" $2 \cdot 3^9 - 5^7$ dollars by giving "B" two 3^9-dollar bills and asking one 5^7-dollar bill from "B" in return.

11. **[10 pts]** For which positive integer n will the equations

$$x_1 + x_2 + x_3 + \cdots + x_{20} = n \quad (1)$$

$$y_1 + y_2 + y_3 + \cdots + y_{68} = n \quad (2)$$

have the same number of positive-integer solutions?