

※請在答案卷內作答

1. (5%)

Find the electrostatic energy required to assemble a sphere of charge of radius  $b$  and the following volume charge density,

$$\rho = \begin{cases} \frac{\rho_0 R}{b}, & 0 \leq R \leq b \\ 0, & R > b \end{cases}, \text{ where } \rho_0 \text{ is a constant.}$$

2. (15%)

A point charge  $Q$  is at a distance  $d$  from the center of a grounded conducting sphere of radius  $a$  ( $a < d$ ). By using the method of images,

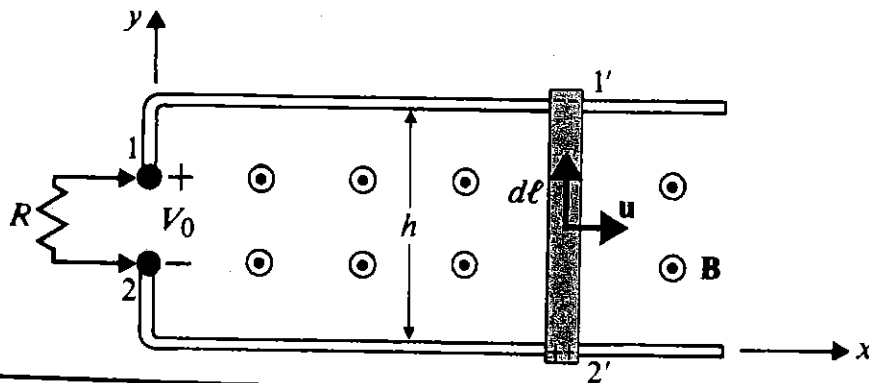
- Find the location and quantity of the image charge  $Q_i$ . (5%)
- Find the charge distribution induced on the sphere surface. (5%)
- Calculate the total charge induced on the sphere. (5%)

3. (20%)

A metal bar slides over a pair of conducting rails in a uniform magnetic field

$\vec{B} = B_0 \cdot \hat{a}_z$ , with a constant velocity  $\vec{u}$ , as shown in following figure.

- Determine the **open-circuit voltage**  $V_0$  that appears across terminals 1 and 2. (5%)
- Assuming that a resistance  $R$  is connected between the terminals, **find the electric power dissipated in  $R$** . (5%)
- Show that this **electric power is equal to the mechanical power** required to move this sliding bar with a velocity  $\vec{u}$ . (Neglect the electric resistance of the metal bar and conducting rails, and all friction). (10%)



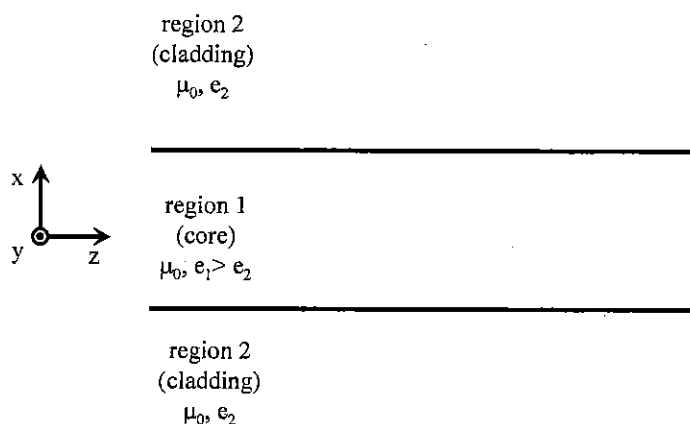
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4. (15%)

Consider a symmetric dielectric slab waveguide where a dielectric slab (core) is surrounded by another dielectric material (cladding) with a lower permittivity as shown below. Assume the wave propagates in  $z$  direction and there is no variation of fields in the  $y$  direction.



- (a) The field components of modes can be solved from the wave equation with boundary conditions. For a mode guided within the slab, what field variations along  $x$  should be considered in the slab and in the cladding? Please also discuss the meaning. (4%)
- (b) Please draw the electric field distribution of the fundamental mode over the transverse plane directly on the figure above and discuss the associated cut-off condition. (3%)
- (c) Assume the core thickness is  $d$  and free-space wavelength is  $\lambda_0$ . If one increases the ratio of  $d/\lambda_0$ , is this waveguide more likely to have more modes or fewer modes? Please explain. (4%)
- (d) When operated from near cut-off to far from cut-off, will a mode be more or less tightly bound to the slab? Please explain. (4%)

5. (15%) Transmission Lines

- (a) Determine the magnitude of the reflection coefficient,  $|\Gamma|$ , of a lossless transmission line connected to a purely reactive load. (5%)
- (b) Find the two shortest lengths of a shorted  $50 \Omega$  lossless transmission line such that their input impedances at 2.25 GHz are identical to that of a capacitor with capacitance  $C_{eq} = 4 \text{ pF}$ . The wave velocity on the line is  $0.75c$ , where  $c = 3 \times 10^8 \text{ m/s}$ . You may express your answers in terms of  $\pi$  and arc-tangents. (10%)

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6. (15%)

Consider a  $z$ -oriented hollow rectangular metallic waveguide with uniform cross section of width  $a$  (along the  $x$  direction) and height  $b$  (along the  $y$  direction), and it is given that  $a > b$ .

- (a) Write down the cutoff frequency for the  $TM_{mn}$  mode and  $TE_{mn}$  mode. (3%, 3%)  
(b) Plot qualitatively typical field lines for  $TE_{10}$  mode within the waveguide on the  $x$ - $y$ ,  $z$ - $y$  and  $z$ - $x$  planes which pass through the center of the waveguide, respectively. Please use solid (dashed) lines for the electric (magnetic) field in your plots. (3%, 3%, 3%)

7. (15%)

A source-free homogeneous vector Helmholtz's equation of E-field can be written as:

$\nabla^2 \mathbf{E} + k_c^2 \mathbf{E} = 0$ , where  $k_c = \omega \sqrt{\mu(\epsilon - j\frac{\sigma}{\omega})}$ . Please define (a) propagation constant, (b)

attenuation constant, (c) phase constant, (d) group velocity, (e) phase velocity, (f) Poynting vector, (g) Brewster angle, and (h) critical angle. (1%, 2%, 2%, 2%, 2%, 2%, 2%, 2%)

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