類組：電機類　科目：離散數學(300C)　

※請在答案卷內作答

一、填空題(共 20 題，每題 3 分：合計 60 分)

答題說明：1. 請**依題號順序**書寫於答案卷，並清楚標註題號 。
2. 題號 1-10 題目詢問內容描述正確與否(題目前標註(T or F)者)。認為描述正確者書寫 T，錯誤者書寫 F。其餘答案一律不給分。
3. 其餘題目(11-20)請直接書寫答案，無需計算過程。

1. The transitive closure of the symmetric closure of the reflexive closure of a relation R is an equivalence relation.

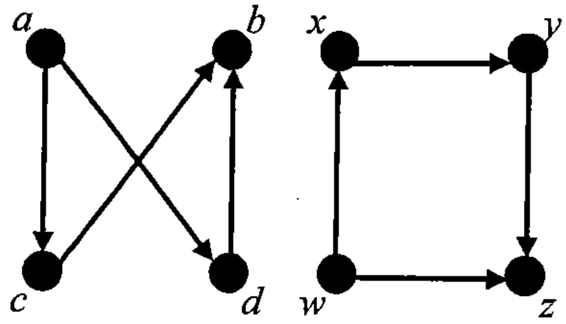
2. The symmetric closure of the reflexive closure of a transitive relation T is an equivalence relation

3. The transitive closure of the reflexive closure of the symmetric closure of a relation R is the smallest equivalence relation that contains R

4. The following incidence matrices are isomorphic:

$$\begin{bmatrix}1 & 0 & 1\\0 & 1 & 1\\1 & 1 & 0\end{bmatrix}, \begin{bmatrix}1 & 1 & 0\\1 & 0 & 1\\0 & 1 & 1\end{bmatrix}$$

5. The given pair of directed graph are isomorphic:



注意：背面有試題

參考用

6. For every Boolean algebra $(K, \cdot, +)$, if a and b in K, then $a \cdot (a + b) = a$ and $a + (a \cdot b) = a$.

7. For every Boolean algebra $(K, \cdot, +)$, if a and b in K, then $a \cdot (\bar{a} + b) = a \cdot b$ and $a + (\bar{a} \cdot b) = a + b$.

For Question 8, 9 and 10, given a connected undirected graph $G = \{V, E\}$ where $|V| = n$ and a positive weight function c on all edges $\{e_i\}$ in E. Suppose Kruskal's algorithm selects $a_1, a_2, \dots, a_{n-1}$ in order to form a spanning tree. Then, consider another tree T with edges $b_1, b_2, \dots, b_{n-1}$ in order such that $c(b_i) \le c(b_{i+1})$ for $1 \le i \le n-1$, decide true (**T**) or false (**F**) for each question:

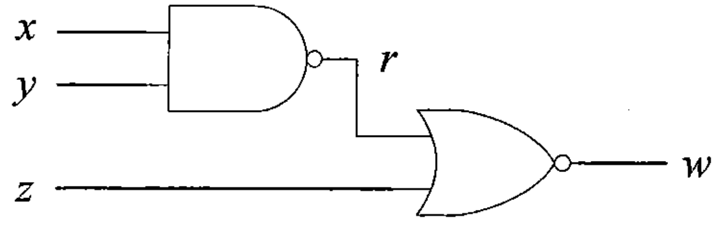
8. $\sum_{i=1}^{n-1} c(a_i) \le \sum_{i=1}^{n-1} c(b_i)$

9. $c(a_i) \le c(b_i),\ \forall i, 1 \le i \le n-1$

10. If T is a minimum spanning tree, then $c(a_i) = c(b_i), \forall i, 1 \le i \le n-1$.

注意：背面有試題

11. Please derive the conjunctive normal form (CNF) for the following circuit:

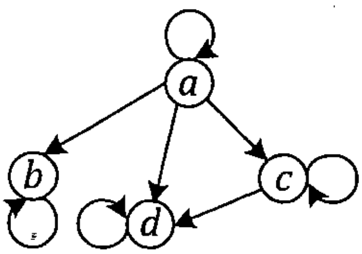


Note that the negated of a variable x should be represented as $\bar{x}$.

12. Solve the following recurrence with the given initial condition:
$a_n = 7a_{n-1} - 12a_{n-2}$ for $n \geq 2, a_0 = 2, a_1 = 1$.

13. Solve the following recurrence with the given initial condition:
$a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2, a_0 = 4, a_1 = 4$.

14. Given a partially ordered graph as follows,



Please draw the corresponding Hasse diagram.

15. How many solutions are there to distribute 8 candies to three children (Tom, John and Mary) in which Tom is given at least 1, John is given at least 2 and Mary is given at least 3 candies?

16. Let G be the grammar with vocabulary $V = \{S, 0, 1\}$, set of terminals $T = \{0, 1\}$, starting symbol S and productions $P = \{S \to 11S, S \to 0\}$. What is $L(G)$ the language of this grammar?

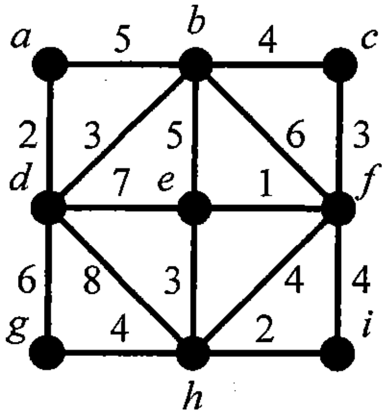
17. Give a phase-structure grammar G that generates the set of all bit strings made up of a 1 followed by an odd number of 0s.

18. Derive a general formula for the recurrences of the form

$$T(n) = \begin{cases} d & \text{if } n \le 1 \\ aT(\frac{n}{b}) + c & \text{otherwise} \end{cases}$$

19. Derive the postfix form of the expression $((x + y) \uparrow 2 + (x - 4)/3)$

20. Draw the minimum spanning tree for the following graph



注意：背面有試題

參考用

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二、問答/計算題(共4大題，每題10分：合計40分)

答題說明：1. 請依題號順序書寫於答案卷，並清楚標註題號。

2. 每題題目描述前說明小題配分。例如：(9分+1分)即代表本題中(a)小題9分與(b)小題1分。以此類推。

21. (5分+5分)

(a) Show that if A is a *context free* set and B is a *regular* set, then $A \times B$ is also context free.

(b) Show that $\{i^2 | i \in \mathbb{N}\}$ is *countably infinite.*

22. (5分+5分)

(a) Show that the conclusion follows from the given premises:

P1: $\bar{Q} \vee P$

P2: $\bar{R} \vee (S \wedge \bar{P})$

P3: $\bar{T} \vee R \vee Q$

C: $S \Rightarrow \bar{T} \vee P$

(b) Show that if $f: X \to Y$ and $g: Y \to Z$ are one-to-one functions, then $g \cdot f: X \to Z$ is also one-to-one function.

23. (5分+5分)

(a) Prove that given n integers, say $x_1, x_2, \ldots, x_n$, there must exist $x_i, +x_{i+1} + x_{i+2} + \cdots + x_{i+k}$ is divided by n for $i \geq 1, k \geq 0$.

(b) Show that if 9 integers are randomly chosen from $\{1, 2, 3, \ldots, 64\}$, there must exist two elements, say x and y, such that

$0 \leq |\sqrt{x} - \sqrt{y}| \leq 1.$

注意：背面有試題

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24. (9 分+1 分)

Suppose you are given a problem of the input size n and the following three algorithms:

- Algorithm X solves problems of size n by dividing them into five subproblems with half the input size, recursively solving each subproblem, and then combining the solutions in linear time.
- Algorithm Y solves problems of size n by recursively solving two subproblems of size n-1, and then combining the solutions in constant time.
- Algorithm Z solves problems by dividing them into nine subproblems of the input size n/3, recursively solving each subproblem, and then combining the solutions in $O(n^2)$ time.

(a) Compute the running times for each of these algorithms in big-O notation.

(b) Which algorithm would you choose?

注意：背面有試題