國立中央大學101學年度碩士班考試入學試題卷

所別：生物醫學工程研究所碩士班 甲組(一般生)　科目：工程數學　共 1 頁 第 1 頁

本科考試可使用計算器，廠牌、功能不拘　＊請在試卷答案卷（卡）內作答

1. Find a general solution of following differential equations (20%)

   (a) $\frac{dy}{dx} + 4y = \frac{4}{3}$

   (b) $x^2\frac{dy}{dx} + x(x+2)y = e^x$

2. Solve the following differential equations using Laplace transformation (20%)

   (a) $2\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 5y = t^2e^{3t}$, subject to $y'(0) = 6, y(0) = 3$

   (b) $\begin{cases} \frac{dy}{dt} + 2\frac{dx}{dt} - 2x = 2 \\ \frac{dy}{dt} + \frac{dx}{dt} - (x+y) = 3 \end{cases}$, subject to $x(0) = 3, y(0) = 3$

3. Convolution:

   (a) Please explain the Convolution theorem using Laplace transformation. (8%)

   (b) Using (a) to evaluate Laplace transform of the following integral equation:

   $y(t) = 2t^2 - \sin(3t) - \int_0^t y(\tau)e^{t-\tau}d\tau$ (10%)

4. Given a matrix A as follows:

$$A = \begin{bmatrix} 3 & 5 & 4 \\ 0 & 2 & 6 \\ 0 & 1 & 3 \end{bmatrix}$$

   (a) Please explain the relationship between eigenvectors and the original matrix (A). (8%)

   (b) Find the eigenvalues of matrix A and the corresponding eigenvectors. (6%)

   (c) Let $x(t) = [x_1(t)\ x_2(t)\ x_3(t)]^T$ and $x'(t) = Ax(t)$ with the initial condition $x(0)=[1\ -2\ 3]^T$. find the solution x(t). (10%)

5. X and Y are independent binomial random variables: X~B(4,0.3) and Y~B(5,0.5).

   (a) Find P(X=2 | X>1). (8%)

   (b) Find P(X=2 | X+Y=5). (10%)