

國立中央大學九十學年度碩士班研究生入學試題卷

所別: 物理學系 不分組 科目: 應用數學 共 1 頁 第 1 頁

1. (12 points) For one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$, find $u(x, t)$ of the string of length π . The initial velocity is zero, and the initial deflection is $\sin 3x$. Please show that the solution can be written in the form of the D'Alembert's solution; that is, the solution is of form $u = \sum_{\pm} f(x \pm t)$.

(3 points) State the physical meanings of your solution.

2. (5 points) Let $z = x + iy$. Find (a) $\text{Im}(1/z)$; (b) $\text{Re}(z^2/\bar{z})$.

(5 points) Represent the following in polar form and plot in the complex plane: (a) $\frac{1-i}{1+i}$; (b) $\frac{7-5i}{4i}$.

(5 points) What is Cauchy integral theorem? What is Cauchy's integral formula?

3. (10 points) Evaluate the integral

$$\int_0^{2\pi} \frac{1 + a \sin \theta}{k + \cos \theta} d\theta, \quad (k > 1)$$

(10 points) Evaluate the integral

$$\oint_C \frac{1}{z^4 - 1} dz$$

counterclockwise over $C: (x - 2)^2 + y^2 = 16$.

4. (10 points) Please find the solution to the equation

$$4x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + \left(x - \frac{1}{36}\right) y = 0$$

(Hint: use $z = \sqrt{x}$ to convert the equation into the Bessel's function)

5. (1) (10 points) Find the inverse of the 3×3 matrix $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & .3 \end{pmatrix}$

(2) (10 points) Please find the determinant and rank of the matrix $\begin{pmatrix} 4 & 3 & 1 \\ -8 & -6 & 2 \\ 16 & 12 & 3 \end{pmatrix}$

6. (1) (10 points) Solve the equation

$$\frac{d^2 y}{dx^2} + 6 \frac{dy}{dx} + 9y = 5 \exp(-x) \cos x.$$

(2) (10 points) Consider a damped harmonic system. Suppose a body with mass m is suspended by a spring with the spring constant k from a ceiling. Assume that when the body is moving, it is subject to a frictional force which is linearly proportional to its moving speed (the coefficient constant is c). Please derive the equation of motion for the body and solve the equation. Consider in particular the three cases: (a) $c^2 > 4mk$, (b) $c^2 = 4mk$, and (c) $c^2 < 4mk$.