[Note : Each problem has been assigned a score. Please attempt as many problems as you can. It is advised to write down details of your procedure step by step.]

1. (20 pts) [Complex Algebra] A particle moves in the (x, y) plane so that its position (x, y) as a function of time t is given by

$$z = x + iy = \frac{i + 2t}{t - i}.$$

(a) Write down complex velocity and complex acceleration. Then find their magnitudes, v and a, as functions of t.

(b) Find x and y as functions of t for the example above, and use these functions to derive v and a again.

2. (20 pts) [Partial Differentiation] The acceleration of gravity can be found from the length l and period T of a pendulum; the formula is $g = 4\pi^2 l/T^2$. Find the relative error in g in the worst case if the relative error in l is 5%, and the relative error in T is 2%.

3. (40 pts) [Multiple Integration] Given a circular plate of radius a and uniform density ρ, find by integration using polar coordinates

(a, 5 pts) its area, A

(b, 10 pts) the centroid of one quadrant $(\bar{x}, \bar{y})$

(c, 10 pts) the moments of inertia, I_x and I_y

(d, 5 pts) its circumference, C

(e, 10 pts) the centroid of a quarter circle arc, $(\bar{x}, \bar{y})$

4. (20 pts) [Multiple Integrations] Consider a transparent disk galaxy, whose luminosity density distribution is given by

$$I(r, z) = I(0, 0) \cdot \exp(-\frac{r}{h_r} - \frac{|z|}{h_z}),$$

where $I(0, 0)$ is the luminosity density at the center, h_r radial scalelength, and h_z is vertical scalelength. Both vertical and radial dimension of this galaxy is infinite. Integrate and derive the total luminosity as a function of $I(0, 0)$, h_r and h_z.