國立中央大學96學年度碩士班考試入學試題卷

所別：光電科學研究所碩士班 一般生 學位在職生 科目：應用數學

1. (8%) Solve the equation $x\frac{dy}{dx} = y - x + 2x^2 - 3x^3$ for y as a function of x.

2. (10%) Given a set of simultaneous differential equations as

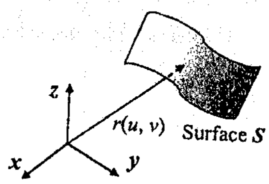
$$\begin{cases} \frac{dI_1(x)}{dx} = -K_1 I_1(x) I_2(x), \\ \frac{dI_2(x)}{dx} = K_2 I_1(x) I_2(x), \end{cases}$$

where K_1 and K_2 are constants. Solve I_1 and I_2 as a function of x under boundary conditions of $\frac{I_1(x)}{K_1} + \frac{I_2(x)}{K_2} = \frac{I_1(0)}{K_1} + \frac{I_2(0)}{K_2} = 1$.

3. (8%) A parametric representation of a surface S in xyz-space is of the form $r(u,v) = x(u,v)\hat{x} + y(u,v)\hat{y} + z(u,v)\hat{z}$, as illustrated by the figure nearby. Show that the area of the surface can be expressed as

$$A_S = \iint [(x_u^2 + y_u^2 + z_u^2)(x_v^2 + y_v^2 + z_v^2) - (x_u x_v + y_u y_v + z_u z_v)^2]^{\frac{1}{2}} \, du dv,$$

where $f_u \equiv \frac{\partial f}{\partial u}$ and $f_v \equiv \frac{\partial f}{\partial v}$ for $f = x, y, z$.



4. (8%) (a). Prove the convolution theorem, i.e.,

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(k) g(k) e^{ikx} dk = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\xi) F(x - \xi) d\xi,$$

where $f(k)$ and $g(k)$ are the Fourier transforms of $F(x)$ and $G(x)$, respectively.

(6%) (b). With (a), prove that the convolution of a Gaussian function of width σ_1 with another Gaussian function of width σ_2 is still a Gaussian function but with a width of $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$ (Hint: A Gaussian function of width σ has the form $G(x) = \frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi}\sigma}$).

5. (10%) Evaluate $\oint_C \frac{dz}{z-2}$ where C is (a) the unit circle, (b) the circle $|z + i| = 3$.

注意：背面有試題

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6. Solve the following initial value problem:

$$\ddot{y} + y = -9\sin(2t)\ ;\quad y(0) = 1\ ;\quad \dot{y}(0) = 0$$ (10%)

7. Find the inverse of the square of the matrix

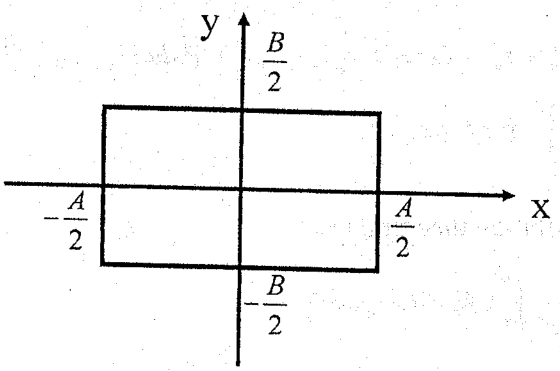
$$\begin{pmatrix} 1 & 2i \\ 3i & 4 \end{pmatrix}$$ (10%)

8. By transforming to a triple integral evaluate

$$I = \iint_S (x^3\, dy\, dz + x^2 y\, dz\, dx + x^2 z\, dx\, dy)$$

Where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ ($0 \le z \le b$) and the circular disks $z = 0$ and $z = b$ ($x^2 + y^2 \le a^2$). (10%)

9. Find the eigenfunctions of the rectangular membrane in the Figure below which is fixed at the boundary. (10%)



10. Evaluate the following real integral:

$$\int_{-\infty}^{\infty} \frac{dx}{(1 + x^2)^2}$$ (10%)