所別：光電科學研究所碩士班　　科目：電磁學

第一題，第二題爲選擇題，可複選，但選錯會倒扣。

Given

$$\nabla \times H = j \tag{1}$$

$$\nabla \times H = j + \frac{\partial D}{\partial t} \tag{2}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{3}$$

$$\nabla \times E = 0 \tag{4}$$

$$\nabla \cdot B = 0 \tag{5}$$

$$\nabla \cdot D = \rho \tag{6}$$

where E is the electric field intensity

D is the electric displacement

H is the magnetic field intensity

B is the magnetic induction

j is the current density

ρ is the free volume charge density

1. (6 points) Which of the above given laws can be derived from the Coulomb's law: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$.

2. (6 points) Which of the above given laws can be derived from the Biot-Savant law:

$$B_p = \frac{\mu_0}{4\pi} I \oint \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

where B_p is the magnetic induction at point P; $\hat{r}$ is the unit positional vector as shown in the figure 1.

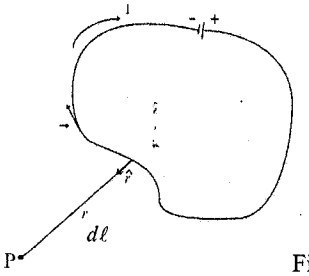

Fig. 1

3. (10 points) It is found in some kind of materials that the direction of the propagation vector $\vec{k}$ of a beam of plane light wave (light waves are a kind of electromagnetic waves) is different from that of its associated Poynting vector $\vec{S}$ (See Fig. 2). Find the equation of the track o f the light ray (光線軌跡之方程式) in the problem.

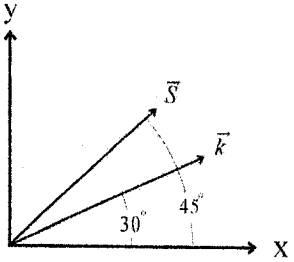

Fig. 2

注意：背面有試題

4. (10 points) Given the wave equation of an electromagnetic field in a conductor as:

$$\nabla^2 E - \varepsilon\mu\frac{\partial^2 E}{\partial t^2} - \sigma\mu\frac{\partial E}{\partial t} = 0$$

where σ is the electric conductivity of the conductor. Show that a plane electromagnetic wave can not propagate in it without decaying (衰減).

5. (18 points) A 40×10^6 Hz plane electromagnetic wave propagates in free space, and its peak electric field intensity is 10.0 millivolts/meter. Calculate the peak voltage induced in a 0.5 meter radius, 10-turn receiving loop oriented so that its plane contains both the electric field and the normal to a wave front. (See Fig. 3)

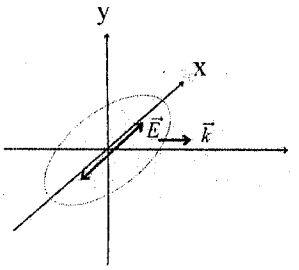

Fig. 3

Given $\varepsilon_0 = 8.85\times10^{-12}$ farad/meter
$\mu_0 = 4\pi\times10^{-7}$ henry/meter

6. (a) (10%) Find the equation for the lines of force in the *xz* plane around an electric dipole $\mathbf{p} = p\hat{\mathbf{e}}_z$ lying along the z-axis. The line of force points in the direction of the electric field, so its differential equation is

$$\frac{dy}{dx} = \frac{E_y}{E_x}.$$

(b) (6%) Sketch a few lines.

7. (10%) A current I of uniform current density flows down a circular cylindrical wire of radius b. Using Ampere's law to find the magnetic field at a distance ρ from the center of the wire, for $\rho < b$ and $\rho > b$.

8. (12%) Two parallel conducting planes located at $z = \pm d/2$ and carry surface current densities $\pm\vec{K} = \pm K\hat{\mathbf{e}}_x$, respectively. Here $K = |\vec{K}|$, and $\hat{\mathbf{e}}_x$ is the unit vector along the *x*-axis. Find the magnetic field everywhere.

9. (12%) Prove that the mean value of the electrostatic potential in vacuum averaged over the surface of a sphere is equal to the potential at the center of the sphere, provided there is no charge inside the sphere.