系所別：光電科學研究所　科目：電磁學

1. (a) Write down the differential form of Maxwell's equations (in terms of $\vec{E},\vec{H},\vec{B}$ and $\vec{D}$ fields) and explain their physical meanings. (5%)

   (b) S is an interface between two media, 1 and 2. The field vectors $\vec{E},\vec{H},\vec{D}$ and $\vec{B}$ (all are time-dependent quantities) at a point on one side of S are related to the field vectors at the neighboring point on the opposite side by boundary conditions. Derive these boundary conditions from Maxwell's equations..
   (Hint: Use divergence theorem and Stokes's theorem). (10%)

   (c) Derive the continuity equation

$$\frac{\partial \rho}{\partial t}+\nabla\cdot\vec{J}=0$$

   from Maxwell's equations and explain its meaning. Here ρ is the free charge density and $\vec{J}$ is the free current density. (5%)

2. For a linear medium (i.e., ε and μ are independent of the field strength), derive

$$\frac{\partial U}{\partial t}+\nabla\cdot\vec{S}=-\vec{J}\cdot\vec{E}$$

   and explain its meaning. (5%)

   Here $U=\frac{1}{2}(\vec{E}\cdot\vec{D}+\vec{B}\cdot\vec{H})$ and $\vec{S}=\vec{E}\times\vec{H}$. (Hint: You can use the vector identity $\nabla\cdot(\vec{E}\times\vec{H})=\vec{H}\cdot(\nabla\times\vec{E})-\vec{E}\cdot(\nabla\times\vec{H})$).

3. f is a scalar function and $\vec{A}$ is a vector field.

   (a) Verify the vector identities. (5%)

$$\nabla\cdot(f\vec{A})=\nabla f\cdot\vec{A}+f\nabla\cdot\vec{A}$$
$$\nabla\times(f\vec{A})=\nabla f\times\vec{A}+f\nabla\times\vec{A}$$

   (b) In spherical coordinate system the gradient of function f is written as

$$\nabla f=\hat{r}\frac{\partial f}{\partial r}+\hat{\theta}\alpha(r)\frac{\partial f}{\partial\theta}+\hat{\phi}\beta(r,\theta)\frac{\partial f}{\partial\phi},$$

   where $\hat{r},\hat{\theta}$ and $\hat{\phi}$ are the unit vectors along the directions of $\nabla r,\nabla\theta$, and $\nabla\phi$, respectively. What is $\alpha(r)$? What is $\beta(r,\theta)$? (5%)

4. Verify that $\psi(x,t)=f(x-ct)+g(x+ct)$ satisfies the 1D wave equation

$$\frac{\partial^2\psi(x,t)}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2\psi(x,t)}{\partial t^2}=0,$$

   Here f and g are two arbitrary single variable functions. (5%)

5. What is Hall effect? (5%) What is its use ? (5%)

6. Charge Q is uniformly distributed on a ring of radius R. This ring is placed on the x-y plane of the x-y-z coordinate system with its center coincide the origin of the coordinate system. A line charge, of length A and with a total charge Q, is situated on the z-axis from z=A to z=2A. Find the electric field intensity at the origin of the coordinate system. (8%)

參考用

注意：背面有試題

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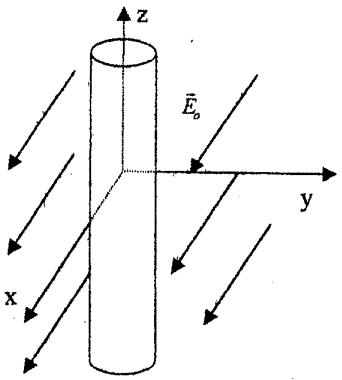
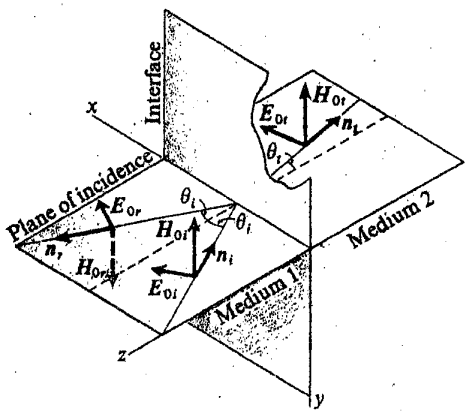
7. A grounded, infinite, cylindrical conductor (radius R) is introduced into a previously uniform electric field with its axis perpendicular to $\vec{E}_o$. Find the potential everywhere outside the cylindrical conductor. You may take the electric field along the x-direction and let the axis of the cylindrical conductor be on the z-axis as shown in the figure below. (10%)

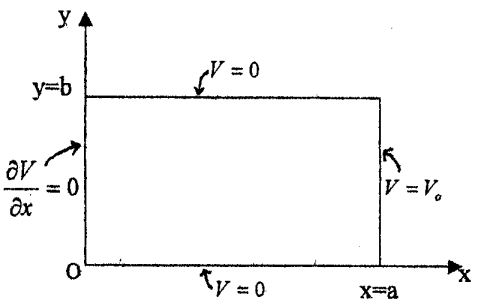


8. A dipole of moment $\vec{p}$ is line up with the z-axis at the origin of coordinates. A second dipole of moment $\vec{p}$ is centered at the point (R, 0, R) and is pointed toward the origin. Calculate the force on the second dipole. (10%)

9. Referring to the figure below, consider a plane wave incident on the boundary of two dielectrics (index of refraction n_1 and n_2, permittivity ε_1 and ε_2, relative permeability $\mu_{r1} = \mu_{r2} = 1$). Suppose that the $\vec{E}$ vector of the wave is parallel to the plane of incidence. Find the Fresnel's equations for both the transmitted wave and the reflected wave. (10%)



10. Try to obtain the potential for any point within the two-dimensional "box" subject to the boundary conditions given in the figure. (12%)



(如有不夠用之變數或參數，請自行定義。)

參考用