參考用

1. a) A, B and C are 3 $n \times n$ matrices, and A is not a zero matrix. Can you conclude from 20%

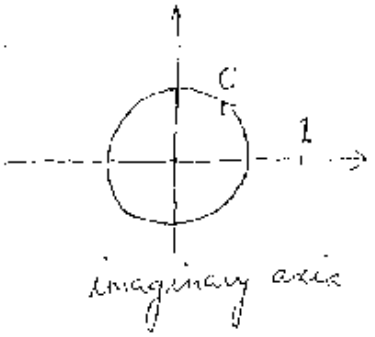
$$A(C - B) = 0$$

that $C = B$. State all possible situations and state clearly their condition.

b) Explain the terms "Wronskian" and "Harmonic functions" respectively.

2. a) Evaluate the following complex integral: 20%

$$\oint_C \frac{dz}{z^2(z-1)}$$



b) Explain the difference between "convergent" and "uniformly convergent" of a power series.

3. Solve the following differential equation $\nabla^2 V = 0$ 20% which satisfies the following boundary conditions:

$$V(x, 0) = 0$$

$$V(x, b) = 0$$

$$V(0, y) = V_0 = \text{constant}$$

$$V(\infty, y) = 0.$$

4. Solve the equation $|x^2 - 1| + 1 = y$ and plot the 20% relation between x and y.

5. Show that

20% a) $div(\vec{u} \times \vec{v}) = \vec{v} \cdot curl\,\vec{u} - \vec{u} \cdot curl\,\vec{v}$

b) $div(g\nabla f \times f\nabla g) = 0$

c) $curl(grad\,f) = 0$

The following formula may be helpful to you

$$\int_0^l \cos\frac{n\pi x}{l}\cos\frac{m\pi x}{l}\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{l}{2} & \text{if } m = n \end{cases}$$