國立中央大學 109 學年度碩士班考試入學試題

所別： 光電類

科目： 電磁學

本科考試可使用計算器，廠牌、功能不拘 *請在答案卷(卡)內作答

Part B

5. (15%) Conservation of charge requires that the volume charge density $\tilde{\rho}_v(\mathbf{r},t)$ at any point in space must be related to the current density $\mathcal{J}(\mathbf{r},t)$ in that neighborhood by the continuity equation

$$\nabla \cdot \mathcal{J}(\mathbf{r},t) + \frac{\partial \tilde{\rho}_v(\mathbf{r},t)}{\partial t} = 0$$

   (a) (10%) Derive this equation from *Maxwell's equations.*
   (b) (5%) In arriving at the above equation, which vector identity must be applied? Please explain it physically.

6. (15%) Consider a plane wave propagating in a simple, nonmagnetic medium characterized by (μ_0, ϵ). Its electric field is given by

$$\mathbf{E}(\mathbf{r}) = (j\mathbf{a}_y - \mathbf{a}_z)\, E_0 e^{-jkx},$$

   where $\mathbf{a}_y$ and $\mathbf{a}_z$ are the unit vectors in the y and z directions, respectively.
   (a) (7%) Determine the magnetic field in vector phasor form.
   (b) (8%) What are the direction of propagation and the polarization state of this plane wave? Explain your answers in as much detail as possible.

7. (20%) An optical beam with a power of 1 W at $\lambda_0 = 532$ nm is normally incident on a crystalline silicon wafer. The reflected power is measured to be 365.7 mW. The surface on the other side of the wafer is perfectly antireflection (AR) coated, as shown in Fig. 3. If this wafer is 10 μm in thickness and the optical power passing through it is 0.16298 mW, answer the following questions in terms of plane wave incidence:

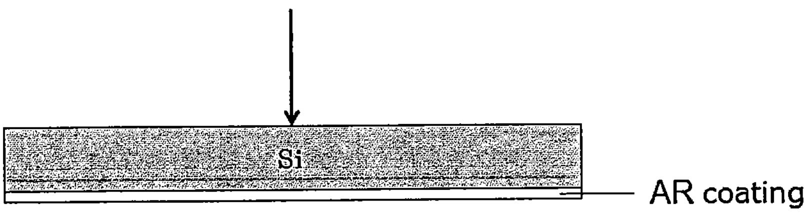

Fig.3

   (a) (10%) Determine the *power* absorption coefficient α of this Si wafer.
   (b) (10%) Determine the complex permittivity of Si at the incident wavelength.

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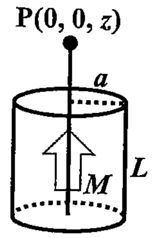
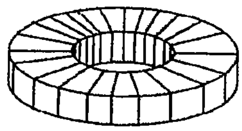
Part A: give the answers in detail (50%):



1. Find the ***magnetic field*** ($\vec{B}$, including its direction and magnitude) on the point P(0, 0, z) of a uniformly magnetized circular cylinder, having a radius a, length L, and magnetization $\vec{M} = M\hat{z}$ with its direction parallel to the axis of the cylinder. (10%)
2. The potential at the surface of a sphere of radius R is $V_0 = k\cos 3\theta$, where k is a constant. Find the ***potential*** inside and outside the sphere. [no charge inside or outside the sphere] (15%)
3. A toroidal coil with the rectangular cross section (inner radius a, outer radius b, and height h), totally carries N turns. (a) Find the ***magnetic field*** ($\vec{B}$, including its direction and magnitude) both inside and outside the toroidal coil. (7%) (b) Define and determine the ***self-inductance*** in terms of a, b, and h. (8%)



4. A sphere conductor of radius a carries a charge Q, and it is surrounded by a linear dielectric material of dielectric constant, ε, out to radius b. Find (a) the ***electric field*** everywhere (5%), and (b) the ***energy*** of this system (5%).

The following formulas may be helpful.

Integral: $\int \frac{dx}{\sqrt{(x^2+y^2)^3}} = \frac{x}{y^2\sqrt{(x^2+y^2)}}$.

Solutions for Laplace's equations by separation of variables:

(a) Cartesian coordinates $V(x,y,z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{n,m} e^{-\pi\sqrt{\left(\frac{n}{a}\right)^2+\left(\frac{m}{b}\right)^2}\,x} \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{m\pi z}{b}\right)$,

(b) Spherical coordinates (azimuthal symmetry) $V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$.

Rodrigues formula: $P_l(x) \equiv \frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l (x^2-1)^l$,

$P_0(x)=1$; $P_1(x)=x$; $P_2(x)=(3x^2-1)/2$; $P_3(x)=(5x^3-3x)/2$; $P_4(x)=(35x^4-30x^2+3)/8$; $P_5(x)=(63x^5-70x^3+15x)/8$.

Fourier's tricks:

$$\int_0^a \sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)dx = \begin{cases} 0, & \text{if } m \neq n \\ \frac{a}{2}, & \text{if } m = n \end{cases},$$

$$\int_{-1}^{1} P_l(x)P_m(x)dx = \int_0^{\pi} P_l(\cos\theta)P_m(\cos\theta)\sin\theta\, d\theta = \begin{cases} 0, & \text{if } m \neq n \\ \frac{2}{2l+1}, & \text{if } m = n \end{cases}.$$

注意：背面有試題