國立中央大學 108 學年度碩士班考試入學試題

所別： 光電類

科目： 電磁學

本科考試可使用計算器，廠牌、功能不拘

Part A: choose the correct answer (50%, 選擇題答案請填於**答案卷**上，非**答案卡**上):

(1) In free space (permittivity: 8.85×10^{-12} F/m), two point charges of $Q_1 = 0.5$ mC and $Q_2 = 2$ μC are respectively located at (x,y,z) = (1,2,0) and $(8,7,\sqrt{7})$ in rectangular (Cartesian) coordinate system, what is the y component of the force acting on Q_2? (A) 5×10^{-2} newtons (B) 5 newtons (C) 0.5 newtons (D) 5×10^{-3} newtons (E) 5×10^{-4} newtons. (5%)

(2) For the above question, what is the magnitude of the force experienced by Q_1? (A) 0.9 newtons (B) 9×10^{-2} newtons (C) 9 newtons (D) 9×10^{-3} newtons (E) 9×10^{-4} newtons. (5%)

(3) A straight wire carrying a current I in the z direction is located along the z axis in free space. If the wire is infinitely long and infinitely thin, what is the magnetic field intensity at the point (0.5L, 0, 0) in rectangular coordinate system? (A) I/πL (B) 2I/πL (C) I/2πL (D) I/4πL (E) I/3πL. (5%)

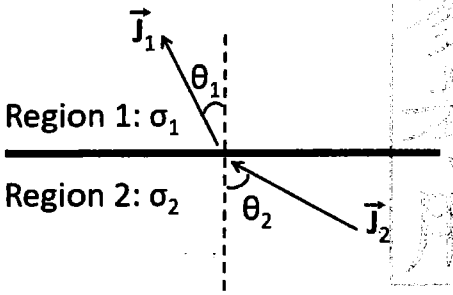
(4) In a perfect dielectric medium with the permeability of μ, there exists an electric field given as $\mathbf{E} = E_0\cos(\omega t - kz)\,\mathbf{a}_x$, where E_0 is the peak value, ω is the frequency, k is a constant, and $\mathbf{a}_x$ is the unit vector in x direction. What is the x component of the magnetic field in the region? (A) $\frac{E_0 k}{\omega\mu}\cos(\omega t - kz)$ (B) 0 (C) $\frac{E_0}{\omega\mu}\cos(\omega t - kz)$ (D) $\frac{E_0 k}{\omega\mu}\cos(\omega t - kz)$ (E) None of the above. (5%)

(5) For an infinite plane carrying uniform surface charge density ρ_s in rectangular coordinate system, what is the magnitude of the electric field intensity at any point on the z axis? (A) $2\rho_s/\varepsilon_0$ (B) $4\rho_s/\varepsilon_0$ (C) 0 (D) ρ_s/ε_0 (E) $\rho_s/2\varepsilon_0$. (ε_0: the permittivity in free space) (5%)

(6) On a metallic sphere of radius R = 30 cm in free space (permittivity: 8.85×10^{-12} F/m), charge Q = 1 μC is uniformly distributed over the surface. What is the electric field just above the surface of the sphere? (A) 1 V/cm (B) 10 V/cm (C) 100 V/cm (D) 1000 V/cm (E) 10000 V/cm. (5%)

(7) Two metallic plates are separated by a distance **d**, and the medium between the plates is homogeneous and has finite conductivity **σ**. If the two plates are of the cross-sectional area **A** and infinite conductivity, what is the resistance of the region between the plates when a potential difference **V** is applied to the two plates? (A) **2d/σA** (B) **V/d** (C) **V/σd** (D) **σV/Ad** (E) None of the above. (5%)

(8) As shown in the figure, a current density vector $\vec{J}$ is passing through an interface between two regions of the conductivities σ_1 and σ_2. If region 1 has the dielectric constant of 2 and the conductivity $\sigma_1 = 40$ μS/m; region 2 has the dielectric constant of 5 and the conductivity $\sigma_2 = 50$ nS/m; $\vec{J}_2$ has a magnitude of 2 A/m^2 and $\theta_2 = 60°$, what is $\vec{J}_1$? (A) 1385.6 A/m^2 (B) 1.732 A/m^2 (C) 1 A/m^2 (D) 1853.7 A/m^2 (E) 1.237 A/m^2. (5%)



(9) For the above question, what is θ_1? (A) 39.96° (B) 89.96° (C) 49.96° (D) 59.96° (E) 69.96°. (5%)

(10) Repeating question (8), what is the surface charge density at the interface? (A) -0.68 mC/m^2 (B) -0.78 mC/m^2 (C) -0.88 mC/m^2 (D) -0.98 mC/m^2 (E) -0.58 mC/m^2. (5%)

注意：背面有試題

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Part B: give the answers in detail (50%):

1. (10%) What are Coulomb and Lorenz gauges? Please check the following two cases, which of them are in Coulomb and/or Lorenz gauges.
   (a) $V = 3t, \quad \vec{A} = A_0 \sin(kx - \omega t)\hat{y}$; where k and A_0 are constants. (5%)
   (b) $V(\vec{r},t) = 0, \quad \vec{A}(\vec{r},t) = -\frac{qt}{4\pi\varepsilon_0 r^2}\hat{r}$, where q and ε_0 are constants. (5%)

2. (10%) You get an extremely thin quarter waveplate, made by a crystal (extraordinary refractive index n_e = 1.671, ordinary refractive index n_o = 1.521, optical axis along x axis), for light with wavelength of 600 nm, as shown in **Figure 1**. (a) Plot the relative direction of the optical axis and the polarization of the light to obtain a circularly polarized light. (4%) (b) Calculate the smallest thickness of the crystal. (6%)

(3) (10%) Please explain what the Poynting vector is. Calculate the power transported down the cable shown in **Figure 2**, using Poynting vector. These two conductors are held at potential difference V and carry current I (down one and back up the other).

(4) (20%) Suppose we have a wave guide of rectangular shape with height a and width b ($a \geqq b$), and we are interested in the propagation of **TM** waves. [**Figure 3**]
   (a) What is a TM wave? Please give the boundary condition, and explain it briefly. (4%)
   (b) Find the fields, $B_x(x,y)$, $B_y(x,y)$ and $B_z(x,y)$. Please give your derivation process. (8%)
   (c) Explain the limitation of m and n in TM_{mn}, where m and n mean the factors of integer for the wave number along x and y directions (k_x and k_y). (4%)
   (d) Derive the cut-off frequency, and explain its physical meaning in detail. (4%)

The components of EM wave in waveguides for E_x, E_y, B_x, and B_y, as well as the uncoupled equations.

$$E_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right), \qquad E_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$

$$B_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right), \qquad B_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(\frac{\omega}{c}\right)^2 - k^2\right]\begin{matrix}E_z \\ B_z\end{matrix} = 0, \text{uncoupled equations}$$

where ω, c, and k are angular frequency, light velocity, and wave number, respectively.

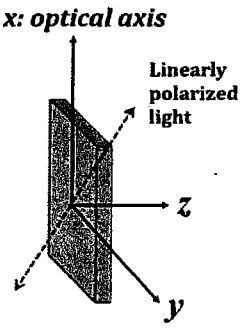

Figure 1

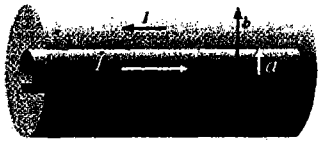

Figure 2

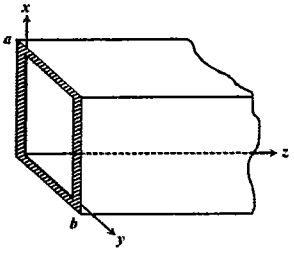

Figure 3

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