國立中央大學106學年度碩士班考試入學試題

所別： 光電科學與工程學系 碩士班 不分組(一般生)　　共1頁 第1頁

科目： 電磁學

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷內作答

1. (25%) Find (a) (15%) the potential and (a) (10%) the electric field inside and outside a uniformly polarized sphere of radius R, as shown in the following figure. $\vec{P}$ is the uniform polarization. [Figure 1]

2. (15%) Find (a) (5%) the magnetic dipole moment of the bookend-shaped loop. All sides have length w, and it carries a current I, as shown in the following figure, (b) (5%) the approximate magnetic field at points far from the origin, and (c) (5%) the approximate magnetic field at points on the $+z$ axis far from the origin. [Figure 2]

3. (10%) Regarding the basic electrostatic properties of ideal conductors, it is well-known that the electric field inside an ideal conductor is zero. Please prove that any net charge resides on the surface. [Hint: Please consider the potential energy.]

4. (10%) The permittivity ε and permeability μ in medium I and II are (ε_1, μ_1) and (ε_2, μ_2), respectively. Assume the free charge density and free current density are both zero in these two media, i.e., $\rho_{free} = 0$, $\mathbf{J}_{free} = 0$.
   (a) (5%) What are the boundary conditions for the $\vec{E}$ and $\vec{H}$ fields on the two sides of the interface between medium I and II ?
   (b) (5%) Explain how can you get these boundary conditions from Maxwell's Equations.

5. (10%) It is known that when total internal reflection happens in the high refractive index medium, non-zero electromagnetic fields still exist in the low refractive index side. Explain why the field energy does not leak out through the interface between the two media.

6. (15%) A point charge moves in a constant velocity.
   (a) (5%) Sketch the electric and magnetic field lines around this charge.
   (b) (5%) Sketch the lines of Poynting vector around the charge.
   (c) (5%) Does this charge radiate any energy? Explain why.
   [Hint: There is no need to calculate anything. Just sketch the filed lines and Poynting vector lines and explain the physics.]

7. (15%) The electric and magnetic fields in free space can be written as $\vec{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$ and $\vec{B} = \nabla \times \vec{A}$, where Φ and $\vec{A}$ are the scalar and vector potentials, respectively.
   (a) (5%) Verify that a gauge transformation does not change the $\vec{E}$ and $\vec{B}$ fields.
   (b) (10%) Suppose the electromagnetic fields are generated by charge density ρ and current density $\vec{J}$, derive the wave equations for Φ and $\vec{A}$.
   (Hint: You can use Lorentz condition $\nabla \cdot \vec{A} = -\frac{1}{c}\frac{\partial \Phi}{\partial t}$ to simplify the final form of the wave equations).

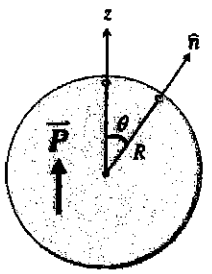

Figure 1

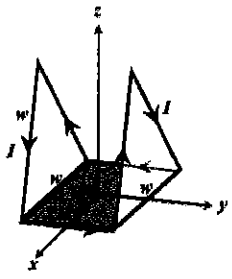

Figure 2

參考用