國立中央大學 105 學年度碩士班考試入學試題

所別： 光電科學與工程學系 碩士班 不分組(一般生)　　共 1 頁 第 1 頁

科目： 工程數學

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷(卡)內作答

10% 1. Solve the initial value problem: $2y'+y=t^2,\ y(0)=0$.

10% 2. Consider an RLC-series circuit with $R=10\ \Omega$, $L=1$ H, and $C=0.25$ F. Find the steady-state current $I(t)$ for an input voltage $V(t)=2\sin(6t)$.

15% 3. Find the general solution of the ODE for $x>0$:

$$x^3y'''+5x^2y''+2xy'-2y=6x.$$

15% 4. A thin wire of length L is made of homogeneous material, has a constant cross section, and is completely thermal insulated laterally, so its temperature distribution $u(x,t)$ is governed by the 1D heat equation: $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$. The wire has the left end ($x=0$) insulated and the right end ($x = L$) kept at zero temperature. Given the initial temperature distribution $f(x)=25\cos\left(\frac{\pi x}{2L}\right)$, solve for $u(x,t)$.

15% 5. In the Cartesian coordinates, the differential Laplacian operator ∇^2 can be expressed as

$$\nabla^2\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

Now, express the differential Laplacian operator ∇^2 in the (u,v,z) parabolic cylindrical coordinating system whose transformation from the Cartesian (x,y,z) is

$$\begin{cases} xy=u \\ x^2-y^2=v \\ z=z \end{cases}.$$

10% 6. Given $K=\begin{pmatrix}0 & 0 & i\\ -i & 0 & 0\\ 0 & -1 & 0\end{pmatrix}$, find the proper choice of n such that $K^n=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$.

7. Diagonalize the matrix $\mathbf{A}$ such that the diagonalized matrix $\mathbf{D}=\mathbf{X}^{-1}\mathbf{A}\mathbf{X}$ is diagonal.

20% (1) Show your details.

$$\mathbf{A}=\begin{pmatrix}-1 & 2 & -2\\ 2 & 4 & 1\\ 2 & 1 & 4\end{pmatrix}$$

5% (2) Calculate $\mathbf{A}^{10}$.