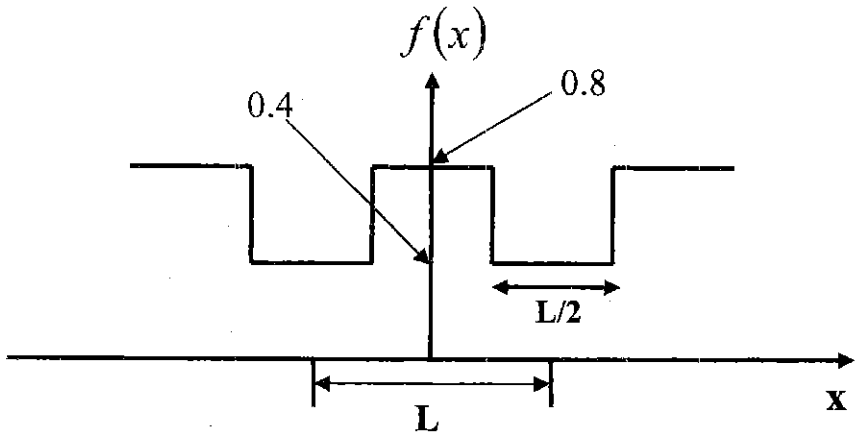
國立中央大學104學年度碩士班考試入學試題

所別：光電科學與工程學系碩士班 不分組(一般生)　科目：工程數學　共 2 頁　第 1 頁

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷（卡）內作答

1. A periodic function $f(x)$ with period L may be expanded as a complex Fourier series $f(x)=\sum_{n=-\infty}^{\infty} a_n\, e^{i2\pi\frac{n}{L}x}$. First, find the equation for a_n (4%) and then use it to expand the following periodic function as a complex Fourier series (6%).



2. (10%) Evaluate $I=\int_0^{2\pi}\frac{d\theta}{3+2\cos\theta}$.

3. (10%) The matrix $\mathbf{C}=\mathbf{AB}$, where $\mathbf{A}=\begin{pmatrix}1 & 2\\ 3 & 0\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}0 & 1\\ 3 & 2\end{pmatrix}$. Find the eigenvalues and eigenvectors of the matrix $\mathbf{C}$.

4. (10%) Find the inverse Laplace Transform of the function $F(s)=\frac{s}{(s+a)(s+b)}$, where $a\neq b$.

5. If a force $\vec{F}$ is given by $\vec{F}=\left(x^2+y^2+z^2\right)^n\left(\hat{i}\,x+\hat{j}\,y+\hat{k}\,z\right)$, where $\hat{i}$, $\hat{j}$, and $\hat{k}$ are the unit vectors in the x, y, and z directions, respectively. Find

(a) (5%) $\vec{\nabla}\cdot\vec{F}$,

(b) (5%) $\vec{\nabla}\times\vec{F}$.

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6. Consider a mass-spring system with $m = 2$ kg and $k = 18$ nt/m.

(a) (3%) Determine the natural frequency of the system in units of rad/s.

(b) (3%) Add a damper so the system is at critical damping. Determine the damping value, including its unit.

(c) (12%) A periodic driving force $4\sin(3t)$ nt is applied to the damped system. Given zero initial displacement from the equilibrium point and zero initial velocity, find the displacement $y(t)$ of the mass as a function of time t.

7. (12%) Find the general solution of the ODE: $4x^2y'' + 2xy' - xy = 0$.

8. Consider a perfectly homogeneous elastic string of length $L = 120$ cm and the speed of wave $c = 400$ m/sec. The string is fastened at both ends $x = 0$ and $x = L$. The initial velocity of the string is zero and the initial deflection (in centimeters) is

$$f(x) = \begin{cases} 0.02\,x, & 0 \le x \le 40; \\ 0.01\,(120 - x), & 40 \le x \le 120. \end{cases}$$

(a) (13%) Find the Fourier series solution of the deflection $u(x,t)$ of the string.

(b) (4%) Sketch the solution $u(x,t)$ for $t = 0$, 1, 2 and 3 msec.

(c) (3%) Determine the frequency (in Hz) of the fundamental mode of the string. What happens to the fundamental frequency if the tension in the string is doubled?

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