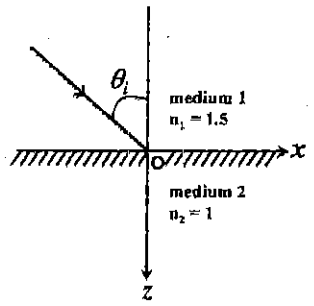
國立中央大學104學年度碩士班考試入學試題

所別：光電科學與工程學系碩士班 不分組(一般生)　科目：電磁學　共 1 頁 第 1 頁

本科考試可使用計算器，廠牌、功能不拘　＊請在答案卷（卡）內作答

(1) (10%) A beam of light is incident from a medium of refractive index 1.5 to another medium of refractive 1 as shown in the accompanying figure. The incident angle is 30° and the wavelength of this beam of light in medium 2 is 0.6 μm. Find the numerical value of k_{2z}, (k_{2z} being the z-component of the wave vector $\bar{k}$ in medium 2) Determine its algebraic sign.



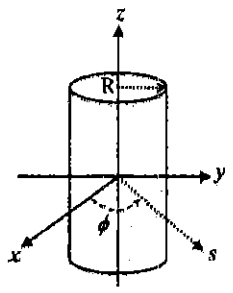
(2) (20%) Repeat problem 1 when the incident angle $\theta_i = 60°$. Also pay attention to the algebraic sign of k_{2z}.

(3) (20%) Consider a waveguide made of perfectly conducting material. The guide is filled with air first (The refractive index of it is taken to be 1), and let the cut-off frequency of a certain mode of TE wave is ω_{ca} (The subscript *ca* stands for cut-off frequency for air). Now, suppose we fill the guide with oil of refractive index 1.5. After this change in the guide, does the cut-off frequency of that particular mode of propagation change? If yes, what is the new cut-off frequency in terms of the old one ω_{ca}? [The following formula may help. $k_g^2 = k^2 - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$, where k is the wave number of a plane wave of infinite cross section propagating in an unbounded space all filled with the medium that is in the guide.]

(4) (5%) Does the given electric field (in spherical coordinates) be an electrostatic field? Explain your answer. $\bar{E}(\bar{r}) = \frac{3k}{r}\left[3\hat{r} + 2\sin\theta\cos\theta\sin\phi\hat{\theta} + \sin\theta\cos\phi\hat{\phi}\right]$

(5) (15%) The potential at the surface of a hollow sphere (radius R) is $k\sin^2(\theta/2)$, where k is a constant. Find the potential inside and outside the sphere, and the surface charge density. [no charge inside or outside the sphere]

(6) (10%) A magnetization, **M**, is carried by a long cylinder (radius R). If $\mathbf{M}=\left(ms^3 + ns^2\right)\hat{\phi}$, where m and n are constants, s is the distance from the axis, and $\hat{\phi}$ is the azimuthal unit vector ($\hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y}$). Find the magnetic field, resulting from **M**, for points outside and inside the cylinder.



(7) (10%) Write down the Maxwell's equations (including the displacement current), and give their names of law, as well as derive the continuity equation (conservation of charge).

(8) (10%) Two individually long cables, with radii a and b (a>b), carry the same uniformly distributed current, I, in one direction over their circularly cross sections. Moreover, the currents return along their surfaces, having very thin insulating sheaths. Find the self-inductances per unit length of these two cables. Which one gets larger self-inductance?