所別：光電科學與工程學系碩士班 不分組(一般生)　科目：工程數學　共 2 頁 第 1 頁

本科考試可使用計算器，廠牌、功能不拘　＊請在試卷答案卷（卡）內作答

17% (1) Find the general solution of the following differential equation:

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=\frac{6}{x}$$

16% (2) Find the general solution of the following differential equation:

$$\frac{d^3y}{dx^3}-2\frac{d^2y}{dx^2}+\frac{dy}{dx}=x$$

7% (3) Consider the following seven differential equations, indicate which of them may have a singular solution and which of them definitely do not. (You don't have to give reason, just indicate.) $f(x)$, $g(x)$, and $r(x)$ are continuous and first differentiable in the interval $-\infty<x<\infty$

a) $y''+f(x)y'+g(x)y=0$

b) $y''+f(x)y'+g(x)y=r(x)$

c) $y''+ay'+by=0$

d) $y''+ay'+by=r(x)$

e) $y'=xy$

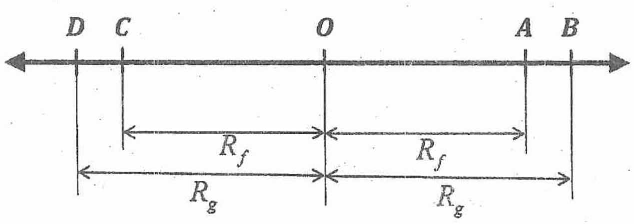
f) $y'=x^2y$

g) $y'=xy^2$

10% (4) Given $y''+f(x)y'+g(x)y=r(x)$, where $f(x),g(x)$ and $r(x)$ are analytic at point O with radii of convergence R_f, R_g and R_r, respectively, as shown in Fig. 1 and $-\infty<R_r<\infty$. Does this equation has a power series solution of the form $\sum_{n=0}^{\infty}c_nx^n$?

If the answer is YES, is this power series solution a solution for $-\infty<x<\infty$ or just for some interval in the x-axis? Indicate that interval.

Fig. 1:



參考用

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(5) Find the normal mode solution of the equations

$$\begin{cases} \frac{d}{dt}x_1 = & -\frac{k}{M}(x_1 - x_2) \\ \frac{d}{dt}x_2 = & -\frac{k}{m}(x_1 - x_2) - \frac{k}{M}(x_1 - x_2) \\ \frac{d}{dt}x_3 = & -\frac{k}{M}(x_1 - x_2) \end{cases} \quad . \qquad (1)$$

10% a) Let $x_j = X_j e^{i\omega t}$, j=1, 2, 3. Substituting into Eq. (1), rewrite the result in a matrix form $\mathbf{AX} = b\mathbf{X}$.

The operator matrix $\mathbf{A} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$, and the vector $\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$.

Derive all α_{mn} (m, n=1, 2, 3) and β.

15% b) Find the eigenvalues of the operator matrix **A** and their corresponding eigenvectors.

10% c) Discover all the normal mode solutions.

(6) In the transformation of two complex numbers $\mathbf{z} = x + iy$ and $\mathbf{w} = u + iv$, where x, y, u, v, and a are real,

$$e^{\mathbf{z}} = \frac{a - \mathbf{w}}{a + \mathbf{w}}, \qquad (2)$$

6% a) Derive the relations among x, y, u, v from the Equation (2). Note: Take the real part and the imaginary, respectively.

3% b) How does the coordinate line $y = 0$ in the **z**-plane transform in the **w**-plane? Draw your result.

3% c) How does the coordinate line $x = 0$ in the **z**-plane transform in the **w**-plane? Draw your result.

3% d) What coordinate system have you constructed on the **w**-plane?

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