所別：光電科學與工程學系碩士班 不分組(一般生)　科目：工程數學　共 2 頁　第 1 頁
　　　光電科學與工程學系碩士班 不分組(在職生)

本科考試可使用計算器，廠牌、功能不拘　　　　＊請在試卷答案卷（卡）內作答

參考用

1. Find the solutions of the following equations:

2% (a) $y'''(x) + 5y''(x) + 3y'(x) - 9y(x) = 0$

2% (b) $x^3y'''(x) + x^2y''(x) - 2xy'(x) + 2y(x) = 0$

3% (c) $y''(x) + y(x) = 5e^x \sin x$

2. Let $\{\phi_n\}$ be the set of orthogonal polynomials that corresponds to the positive weight function $w(x)$ on the finite interval (a,b).

$$\int_a^b w(x)\cdot\phi_n(x)\cdot\phi_k(x)\cdot dx = \begin{cases} C_n & n=k \\ 0 & n\neq k \end{cases},\ \text{where } C_n \text{ is a constant.}$$

Let w be of the form $w(x) = (x-a)^\alpha (x-b)^\beta$, where α>-1, β>-1.

5% (a) Evaluate $\int_a^b \frac{d}{dx}[(x-a)(x-b)\phi'_n(x)w(x)]Q(x)dx$,

for every polynomial $Q(x)$ of degree less than n.

8% (b) Show that

$$(x-a)(x-b)\phi''_n + [(2+\alpha+\beta)x - a(1+\beta) - b(1+\alpha)]\phi'_n = A_n\cdot\phi_n.$$

Also find the the proportional constant A_n.

9% 3. You are given

(a) A set of functions $u_n(x)=x^n$, n=0, 1, 2,...

(b) An interval $(0,\infty)$,

(c) A weighting function $w(x) = xe^{-x}$.

Construct the first three **orthonormal** functions from the set $u_n(x)$ for this interval and this weighting function.

5% 4. The Legendre polynomials $P_n(x)$ are orthogonal with respect to the weight function $w(x)=1$ on the ontervl (-1,1) and

$$\|P_n\|^2 = \int_{-1}^1 [P_n(x)]^2 dx = \frac{2}{2n+1},\quad \text{where } n\geq 0.$$

$$P_0(x) = 1,\quad P_1(x) = x,\quad P_2(x) = \frac{1}{2}(3x^2-1),\ \dots\ .$$

Therefore, the coefficients in the Fourier-Legendre series,

$$\sum_{n=0}^{\infty} C_n P_n(x)$$

for an arbitrary function $f(x)$ are given by

$$C_n = \frac{2n+1}{2}\int_{-1}^1 f(x)P_n(x)\,dx,\quad n\geq 0.$$

Now, it is given that

$$f(x) = \begin{cases} x, & 0\leq x\leq 1 \\ 0, & -1\leq x\leq 0. \end{cases}$$

Find the Fourier-Legendre series for $f(x)$.

5. Find the directional derivative of $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ at $P:(1,1,1)$ in the direction $\vec{a} = [1,2,0]$. (9%)

6. Find the eigenvalues and the corresponding eigenvectors for the matrix $\begin{bmatrix} 2 & 7 \\ 6 & -9 \end{bmatrix}$. (12%)

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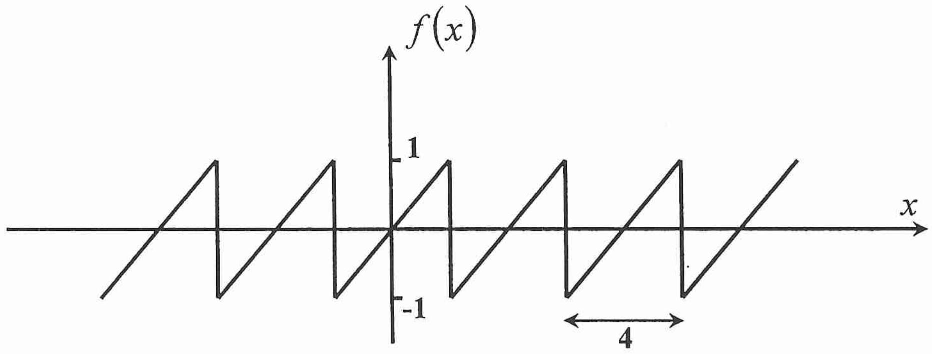
7. A periodic function $f(x)$ may be expanded as the complex Fourier series: $f(x)=\sum_{n=-\infty}^{\infty} c_n e^{i2\pi\frac{n}{L}x}$,

where L is the spatial period. You have to find, first, the general expression for c_n and then apply it to find the "**complex Fourier series**" for the function $f(x)$ shown below. (12%)



8a) Consider the following equation $\dfrac{\frac{\partial}{\partial x} f_1(x)}{f_1(x)} + \dfrac{\frac{\partial}{\partial y} f_2(y)}{f_2(y)} = constant$,

where $f_1(x)$ and $f_2(y)$ are regarded as a real valued function of x and y. Are the

$\dfrac{\frac{\partial}{\partial x} f_1(x)}{f_1(x)}$ and the $\dfrac{\frac{\partial}{\partial y} f_2(y)}{f_2(y)}$ necessarily a constant of x and y? Give reason. (6%)

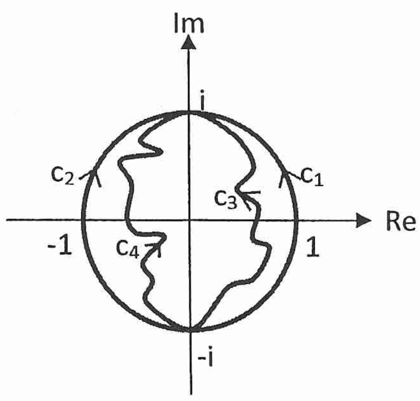
8b) Consider the following equation $\dfrac{\frac{\partial}{\partial x} f_1(x,y)}{f_1(x,y)} + \dfrac{\frac{\partial}{\partial y} f_2(x,y)}{f_2(x,y)} = constant$,

are the $\dfrac{\frac{\partial}{\partial x} f_1(x,y)}{f_1(x,y)}$ and the $\dfrac{\frac{\partial}{\partial y} f_2(x,y)}{f_2(x,y)}$ necessarily a constant of x and y? Give reason. (7%)

9) Given a complex function $\frac{1}{z}$, find the definite integral $\int_{-i}^{+i} \frac{1}{z} dz$

a) along curve C_1 of the semi-circle centering at (0,0) and of radius equal to 1. The direction of the integration is counterclockwise.(See the accompanied figure) (5%)

b) along curve C_2 on the left side of the semi-circle in the figure. The direction of the integration is clockwise this time. (5%)



c) Will the result of this definite integral have the same value as part (a) or part (b) if the integration limits remain the same but the path and the direction of integration is changed to C_3 as shown in the figure. Give reason regardless of the answer is Yes or No. And, if the answer is No, can you find its value? (5%)

d) Repeat part (c) with curve C_4 and the direction of integration as indicated in the figure. (5%)

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