國立中央大學100學年度碩士班考試入學試題卷

所別：光電科學與工程學系碩士班 不分組(一般生)　科目：電磁學　共 2 頁 第 1 頁

本科考試可使用計算器，廠牌、功能不拘

*請在試卷答案卷（卡）內作答

參考用

1. An electrostatic mop is a very common household cleaning tool nowadays as shown in Figure 1. It utilizes the electrostatic charge built up in microfibers to attract and remove dust particles on the floor. The charged area usually is rectangular in shape with the short and long edges of a and b. If the total charge built up on the surface of the mop is Q and the distribution of the charge is uniform over the entire surface. Please answer the following questions.

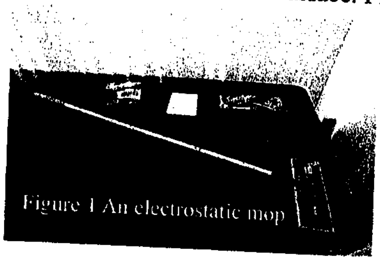

Figure 1 An electrostatic mop

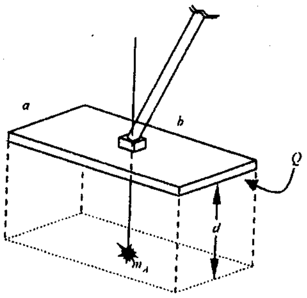

Figure 2 The electrostatic mop and a dust particle on the floor

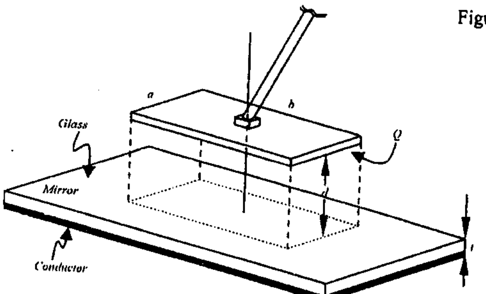

Figure 3 The electrostatic mop is place on top of a big mirror

Part A. Typical floors are insulators. The mop has a distance of d from the floor and the mop surface aligns parallel to the floor. The dust size is very small compared with the mop area.

a. (5 pts) If a dust particle with total charge of q and mass of m_A is exactly at the projected center of the mop as Figure 2 shows. Please derive the condition that the dust particle can be removed by the mop without considering the floor and whatever effects it may cause.

b. (4 pts) Floor and dust particle can be dielectric material or conductors. Sketch the electric force lines and charge distribution in space of (1) Conductor floor-conductor dust, and (2) dielectric floor-dielectric dust. (Note: All the dielectric materials has relative dielectric constant larger than 1.)

c. (8 pts) A conductive dust particle which is a cubic in shape with each side of u in length. The particle does not have initial charge before the mop gets close. The dimension of the particle is far smaller than the mop. If the floor has relative dielectric constant equal to 1. Please derive the induced dipole moment of this dust particle. Will the dust particle be remove or not? Describe and explain your answer. (Hint: Please consider different forces and how are the charges interact. Induced dipole has to be considered.)

Part B. Mirror is usually made of a slab of glass coated with highly reflective metal on the back. Highly reflective metal implies the metal is almost a perfect conductor. Assume we have a big mirror which has glass thickness of t. The glass dielectric constant is ε.(Note: Not relative dielectric constant.) As in Figure 3, we place the electrostatic mop on top of the mirror surface with a distance d. Answer the following question without considering the fringing field.

a. (3 pts) What is the electric field in the air between the glass and the mop?

b. (5 pts) What is the polarization of the glass slab?

c. (5 pts) What is the capacitance of the entire system?

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2. The orbit of a geostationary satellite is situated right over the equator by 35000 km. To keep the antenna system properly oriented, attitude control can be achieved by means of coils whose magnetic fields interact with that of the Earth.

a. (10pts) Calculate the number of ampere-turns required for a coil wound around the outside surface of a satellite whose diameter is 1 meter. Assume that the required torque is 10^{-3} Newton-meter, and the magnetic field at the altitude of the satellite orbit is 5×10^{-5} Tesla. The angle between the earth's magnetic field and the normal to the coil is 10 degrees.

b. (5pts) What happens if the satellite is situated at the zero declination line? And what happens when the magnetic field of the earth is reversed?

3. a. (5pts) Show that the coordinate free form of the magnetic field of a dipole can be expressed as

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$$

b. (5pts) If a magnetic dipole levitating above an infinite superconducting plane is free to rotate as shown in Fig. 4, what orientation will it adopt? (Hint: The magnetic field inside superconductors is zero. You may use method of images to solve this question.)

c. (5pts) How high above the surface will the dipole float? (Assume that the mass of the dipole is M, and the gravity constant is g.)

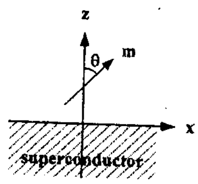

Fig. 4

4. (13pts) When treating the problem of refraction and reflection of electromagnetic waves striking an interface of two media, usually we consider the parallel component and the perpendicular component (to the incident plane) of the E-field of the incident waves separately. Show that :

$$\langle S \rangle_T = \langle S \rangle_P + \langle S \rangle_N$$

Where $\langle S \rangle_T$ is the average Poynting vector of the incident light beam, $\langle S \rangle_P$ and $\langle S \rangle_N$ being the corresponding average Poynting vector of the parallel component and normal component, respectively, of the incident beam.

5. (14pts) A beam of light waves is made to pass through a finite open aperture as shown in Fig. 5. After passing through the aperture, does the cross section of this light beam have exactly the same shape of the open aperture with infinitely definite boundary? Discuss your result from the point of view of electromagnetic theory.

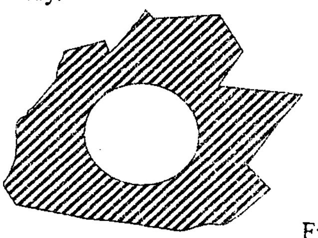

Fig. 5

6. (13pts) State the kind of boundary conditions that one would use to find the characteristic modes of both TE and TM electromagnetic waves traveling along a perfectly conducting wave guide. Specifically, what physical quantities are involved in these boundary conditions and what are their mathematical expressions?

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