

國立中央大學九十學年度碩士班研究生入學試題卷

所別: 統計研究所 不分組 科目: 基礎數學 共 1 頁 第 1 頁

1. Find $\int_0^1 \frac{x^b - x^a}{\log x} dx$, where $a, b > 0$. (10%)

2. Find $\lim_{n \rightarrow \infty} \frac{\sqrt[n]{n!}}{n}$. (10%)

3. Find a polynomial function $f(x)$ with the smallest degree so that $\lim_{x \rightarrow 1} \frac{f(x)}{x-1} = 1$,
 $\lim_{x \rightarrow 2} \frac{f(x)}{x-2} = 2$ and $\lim_{x \rightarrow 3} \frac{f(x)}{x-3} = 3$. (10%)

4. Find $\int \frac{(\log x)^2}{\sqrt{x}} dx$. (10%)

5. Find $\int_0^\infty \int_0^x x^{-3/2} e^{y-x} dx dy$. (10%)

6. Let $A = \begin{pmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{pmatrix}$, a, b, c, d are real numbers, find the determinant of A . (10%)

7. Let $A = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$, Find A^n , n positive integer. (10%)

8. Let A be an $n \times n$ real matrix with eigenvalues $1 > \lambda_1 > \lambda_2 > \dots > \lambda_n > 0$. Show that $\lim_{m \rightarrow \infty} A^m \underline{x} = 0$ for $\underline{x} \in R^n$. (10%)

9. Let V be a vector space spanned by the vectors $(1, 1, -1, -1)^t$, $(1, 0, 0, -1)^t$ and $(1, 1, -3, 1)^t$. Find the projection of $(1, 2, 3, 4)^t$ onto V . (10%)

10. Let $T: R^2 \rightarrow R^2$ be a linear transformation such that $T((x_1, x_2)^t) = (2x_1 - x_2, 3x_1 + 2x_2)^t$

(a) Find the matrix A associated with T , when the basis is the standard basis. (5%)

(b) If $\beta = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$ and $\beta' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$, find the matrix B associated with T , depending on the bases β and β' . (5%)