I  Let $X_1, ..., X_n$ be a random sample from the normal distribution with mean 0 and variance $\theta^2$, where $0 < \theta < \infty$.

   (a)  Find the maximum likelihood estimator (MLE) for $\theta$, denoted by $\hat{\theta}$.  (10%)

   (b)  Show that the MLE is consistent for $\theta$.  (8%)

   (c)  Find the function $\gamma(\theta)$ so that the asymptotic distribution of $\sqrt{n} (\hat{\theta} - \theta)/\gamma(\theta)$ is a standard normal as $n \to \infty$.  (10%)

II  Let $X_1, ..., X_n$ be a random sample from the Poison distribution with parameter $\lambda$.

   (a)  Find the uniformly minimum variance unbiased estimator (UMVUE) of $(1+\lambda)e^{-\lambda}$, the probability of $X_1 = 0$ or $X_1 = 1$.  (15%)

   (b)  Find a 95% confidence interval for $\lambda$.  (10%)

III  Let $\lambda$ be the proportion of defective items in a batch of products. For testing $H_0: \lambda=0.05$ versus $H_1: \lambda>0.05$, we take a random sample of 100 items from the batch. Let Y denote the number of defective items in the sample.

   (a)  If we observe $Y = 10$. What is the associated p-value ? Draw your conclusion according to the p-value.  (7%)

   (b)  If we want to estimate $\lambda$ within an error of 0.05, justity whether the sample size of 100 is enough to guarantee such an estimation with probability 0.95?  (5%)

IV  Let $X_1$ and $X_2$ be a random sample from the distribution with probability density function $\lambda e^{-\lambda x}$ for $x > 0$. We consider to reject $H_0: \lambda=\log 2$ in favor of $H_1: \lambda< \log 2$ if $X_1 + X_2 \geq c$.

   (a)  Find the value of c so that the size of the test is 0.05.  (10%)

   (c)  Find the power of the size 0.05 test at $\lambda=2\log 2$.  (10%)

V  Let $X_1, ..., X_n$ be a random sample from the distribution with probability density function $e^{-(x-\theta)}$ for $\theta \leq x < \infty$, where $-\infty < \theta < \infty$. Find the size $\alpha$ likelihood ratio test of $H_0: \theta=0$ versus $H_1: \theta \neq 0$. Specify clearly the associated critical value.  (15%)