國立中央大學八十五學年度碩士班研究生入學試題卷

所別：統計研究所　甲組　　科目：數理統計　　

1. Let $X_1, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$, to estimate σ^2
   (a) If μ is unknown, let $S_1^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$, $S_2^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$ and $S_3^2 = \frac{1}{n+1}\sum_{i=1}^{n}(X_i - \overline{X})^2$, where $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Which one is best ? Why ? (10%)
   (b) If μ is known, can you find a better estimator ? (10%)
2. Let $X_1, \cdots, X_n$ be a random sample from uniform distribution on $(0, \theta), \theta > 0$, show that $T_n = e(\prod_{i=1}^{n} X_i)^{\frac{1}{n}}$ is a consistent estimator of θ. (20%)
3. Let $X_1, \cdots, X_n$ be i.i.d. with density function $f(x) = \lambda e^{-\lambda x}, x > 0, \lambda > 0$. Find a $100(1-\alpha)\%$ confidence interval for λ. (20%)
4. Let $X_1, \cdots, X_n$ be i.i.d. Poisson distribution with mean μ. Find a good test for testing the hypothesis $H_0 : \mu = 7$ v.s. $H_1 : \mu \neq 7$ with significance level α. (20%)
5. Let $X_i \sim N(\mu, \sigma_i^2), i = 1, \cdots, n$ be independent ($\sigma_1^2, \cdots, \sigma_n^2$ may not be all equal). If $Y = \sum_{i=1}^{n} \frac{X_i}{\sigma_i} / \sum_{i=1}^{n} \frac{1}{\sigma_i}$, $Z = \sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma_i} - \frac{Y-\mu}{n}\sum_{i=1}^{n}\frac{1}{\sigma_i}\right)^2$, show that
   (a) Y and Z are independent. (10%)
   (b) $Z \sim \chi^2_{n-1}$. (10%)