

國立中央大學八十四學年度碩士班研究生入學試題卷

所別：統計研究所 甲組

科目：機率論

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1. Let X_1, X_2, X_3 and X_4 be independent $N(0, 1)$. $Y_1 = \sqrt{X_1^2 + X_2^2}$, $Y_2 = 2\sqrt{X_3^2 + X_4^2}$. Find $P(Y_1 > Y_2)$. (20%)
2. Let X be a random variable with density function $f(x) = \frac{4x^2}{\sqrt{\pi}}e^{-x^2}$, $x > 0$. Find (a) $E(X)$ (b) $Var(X)$ (c) $E(X^2)$ and (d) $Var(X^2)$ (5% each part)
3. Use integration by parts and central limit theorem to find $\lim_{n \rightarrow \infty} \frac{1}{n!} \int_0^n t^n e^{-t} dt$. (20%)
4. Let (X, Y) be a random vector with joint density
$$f(x, y) = \frac{6}{5}(x^2 + y), \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1,$$
and $Z = \begin{cases} 2X^2 & \text{if } X \leq Y \\ 3XY & \text{if } X > Y \end{cases}$.
Find (a) $E(Z|X = x)$ and (b) $E(Z)$ (10% each part)
5. Let X and Y be independent random variables, $X \sim N(0, 1)$, $P(Y = 1) = P(Y = -1) = 1/2$, and $Z = XY$. Show that
 - (a) $Z \sim N(0, 1)$, (10%)
 - (b) $Cor(X, Z) = 0$, (5%)
 - (c) X and Z are not independent. (5%)