

1. Find  $\lim_{n \rightarrow \infty} \frac{1}{(2n+1)^2} \left[ \frac{(4n+1)!}{(2n+3)!} \right]^{\frac{1}{n}}$ . (10%)
2. Let  $f(x) = \ln(1+x)$ .
- (a) Find the fourth-degree Taylor polynomial for  $f$  expanded about  $x_0 = 0$ , and use it to approximate  $\ln(1.1)$ . (13%)
- (b) Find a bound for the error in the approximation in (a). (7%)
3. Find an approximate root of  $x^3 - 2x^2 - 5 = 0$  in  $[1, 4]$  with  $10^{-4}$  accuracy by Newton's method. (10%)
4. Let  $F(x) = \int_{-x^2}^{e^x} \frac{1}{1+t^2} dt$ . Find  $F'(1)$ . (10%)
5. Find the solution of the initial value problem:  
 $x^2 y'' - xy' + y = 0$ ,  $y(-1) = 1$ ,  $y'(-1) = 0$ . (10%)
6. Find an orthonormal basis in  $R^3$  for the set of vectors in the plane  $2x - y - z = 0$ . (10%)
7. Let  $A = \begin{pmatrix} -1 & -3 & -9 \\ 0 & 5 & 18 \\ 0 & -2 & -7 \end{pmatrix}$
- (a) Find the eigenvalues of  $A$  and the associated eigenvectors. (12%)
- (b) Find the eigenvalues of  $A^{-1}$  and the associated eigenvectors. (5%)
- (c) Is  $A$  positive semidefinite? State your reason. (5%)
- (d) Find  $A^{-1}$ . (8%)

參考用