

國立中央大學 111 學年度碩士班考試入學試題

所別： 統計研究所 碩士班 不分組(一般生)

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統計研究所 碩士班 不分組(在職生)

科目： 基礎數學

計算題應詳列計算過程，無計算過程者不予計分

1. (10%) Determine

$$\lim_{n \rightarrow \infty} \frac{1 + 1/2 + \cdots + 1/n}{\ln n}.$$

2. (10%) Determine the first digit after the decimal point of the following integral value.

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{x - \sin(x)}{x^3} \exp\left(-\frac{x^2}{2}\right) dx.$$

3. (20%) Let $F(x) = \exp(-\exp(-x))$. Find a_n so that $F(a_n) = 1 - n^{-1/2}$ and determine α such that $\lim_{n \rightarrow \infty} [\exp(a_n)]/n^\alpha$ converges to a nonzero constant.

4. (20%) Suppose $f(x)$ is a differentiable on \mathbb{R} with $1 \leq |f''(x)| \leq 2$ for all $x \in \mathbb{R}$. Let $g(x) = |x|$ if $x \in [-1, 1]$; $g(x) = 0$ otherwise. Prove that

$$\left| \frac{1}{h} \int_{-\infty}^{\infty} f(u) g\left(\frac{u-x}{h}\right) du - f(x) \right| < \frac{1}{2} h^2 \text{ for each } h > 0.$$

[Hint: let $v = (u-x)/h$ and consider the Taylor's expansion of $f(x+hv)$ at x .]

5. Let 3×2 matrix $V = \begin{pmatrix} 2/3 & 2/3 \\ 2/3 & -1/3 \\ 1/3 & -2/3 \end{pmatrix}$ and let H be a 3×3 matrix such that $H = I_3 - VWV^T$,

where I_3 is the 3×3 identity and W is a 2×2 matrix.

(a) (10%) Find three different W s such that $H^2 = H$.

(b) (10%) Show that there is no W such that $H^2 = \alpha H$ if $\alpha \neq 1$.

6. (20%) For $x, y \in \mathbb{R}$, let $\phi(x, y) = \exp(-(x^2 + 4xy + y^2)) + 2(x + y)$. Find the maximum and minimum of $\phi(x, y)$ on the region $x \geq 0, y \geq 0$, and $x + y \leq 1$.