國立中央大學 106 學年度碩士班考試入學試題

所別： 統計研究所 碩士班 不分組(一般生)　　共 1 頁　第 1 頁

統計研究所 碩士班 不分組(在職生)

科目： 基礎數學

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷　內作答

1. [60%] Let $f(x)=\begin{cases}(7x^3-18x^2+12x)/2 & \text{if} \quad 0\le x<1\\ -(x-2)^3/2 & \text{if} \quad 1\le x\le 2\end{cases}$ be a function defined on $[0,2]$.

(a) [10%] Find a critical point of $f(x)$.

(b) [5%] Prove that $f(x)$ is continuous at $x=1$ [check the left- and right- hand limits].

(c) [5%] Calculate the minimum, $f_{\min}\equiv\min_{x\in[0,2]} f(x)$.

(d) [10%] Calculate the maximum, $f_{\max}\equiv\max_{x\in[0,2]} f(x)$.

(e) [10%] Sketch the graph of $f(x)$. The graph must include the locations of $f_{\min}$ and $f_{\max}$ and the locations of $f''(x)>0$, $f''(x)<0$, and $f''(x)=0$.

(f) [10%] Calculate $\int_0^2 f(x)dx$.

(g) [10%] Calculate $\int_0^2\{f''(x)\}^2dx$.

2. [20%] Let λ_i, $i=1,...,p$, be the eigenvalues of a symmetric and positive definite matrix A.

(a) [10%] Write $\text{tr}(A)$ in terms of λ_i, $i=1,...,p$. Here $\text{tr}(A)$ is the trace of A.

(b) [10%] Write $|A|$ in terms of λ_i, $i=1,...,p$. Here $|A|$ is the determinant of A.

參考用

3. [20%] Let $A=\begin{bmatrix}16 & -11 & 2 & 1 & 0\\ -11 & 8 & -2 & -1 & 1\\ 2 & -2 & 2 & -2 & 2\\ 1 & -1 & -2 & 8 & -11\\ 0 & 1 & 2 & -11 & 16\end{bmatrix}$ and $\mathbf{x}=\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{bmatrix}$.

(a) [5%] Calculate $\mathbf{x}^{\mathrm{T}}A\mathbf{x}$ when $\mathbf{x}^{\mathrm{T}}=(1,1,1,1,1)$. Here "T" denotes the transpose of $\mathbf{x}$.

(b) [5%] Calculate $\text{tr}\left\{\begin{bmatrix}16 & -11 & 2 & 1 & 0\\ -11 & 8 & -2 & -1 & 1\\ 2 & -2 & 2 & -2 & 2\\ 1 & -1 & -2 & 8 & -11\\ 0 & 1 & 2 & -11 & 16\end{bmatrix}\begin{bmatrix}1 & 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1 & 1\end{bmatrix}\right\}$.

(c) [10%] Calculate $\left.\dfrac{\partial\mathbf{x}^{\mathrm{T}}A\mathbf{x}}{\partial x_1}\right|_{\mathbf{x}=(1,1,1,1,1)}$.