國立中央大學106學年度碩士班考試入學試題

所別： 統計研究所 碩士班 不分組(一般生) 共1頁 第1頁
統計研究所 碩士班 不分組(在職生)

科目： 數理統計

本科考試可使用計算器，廠牌、功能不拘 須有計算過程 ＊請在答案卷內作答

參考用

1. Suppose that the pair of random variables (X, Y) has the joint density

$$f(x,y) = \frac{\Gamma(\alpha+\beta+\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)} x^{\alpha-1} y^{\beta-1} (1-x-y)^{\gamma-1},$$

for $x > 0$, $y > 0$, and $x + y < 1$.

   (a) (10%) Find the joint density of $S = X + Y$ and $R = \frac{X}{X+Y}$.

   (b) (10%) Assume that α and β are known. From n independent and identically distributed copies of the pair (X_i, Y_i), show that $\Pi_{i=1}^n (1 - S_i)$ is a complete sufficient statistic for estimating γ and $\Pi_{i=1}^n R_i$ is an ancillary statistic for estimating γ. What does Basu's theorem say in this context?

2. Suppose $Y_1, Y_2, \cdots, Y_n$ are independent random variables with probability density functions (pdf for short) written as

$$f_i(y_i) = \beta_i e^{\beta_i y_i} \quad y_i \geq 0$$

where $\beta_i = \theta x_i$ for unknown parameter $\theta > 0$ and fixed unknown constants $x_i > 0$ for $i = 1, 2, \cdots, n$.

   (a) (10%) Show that the joint pdf $f(y_1, \cdots, y_n|\theta)$ forms a one parameter exponential family with minimal sufficient statistic $T = \sum_{i=1}^n x_i Y_i$.

   (b) (10%) What is the probability distribution of T?

   (c) (10%) What is the exact (if possible) or an approximated confidence interval with confidence coefficient $1 - \alpha$ for θ?

3. Let $(X_1, \cdots, X_n)$ be a random sample from a population with the probability density function f. Let θ_0 and θ_1 be two constants with $\theta_0 < \theta_1$. Obtain a size α uniformly most powerful test for testing $H_0 : \theta = \theta_0$ versus $H_a : \theta = \theta_1$ in the following cases

   (a) (10%) $f(x) = e^{-(x-\theta)}$ for $x \geq \theta$.

   (b) (10%) $f(x) = \theta x^{-2}$ for $x \geq \theta$.

4. Let $(X_1, \cdots, X_n)$ be a random sample with the probability density f. Find a maximum likelihood estimator of θ in the following cases

   (a) (10%) $f(x) = \frac{1}{\sqrt{2\pi\theta^2}} e^{\frac{(x-\theta)^2}{2\theta^2}}$ for $-\infty < x < \infty$ and $\theta \neq 0$.

   (b) (10%) $f(x) = \theta^x (1-\theta)^{1-x}$ for $x = 0$ or $x = 1$ and $\theta \in [\frac{1}{6}, \frac{5}{6}]$.

   (c) (10%) $f(x) = 1$ for $0 < x < 1$ if $\theta = 1$ and $f(x) = \frac{1}{2\sqrt{x}}$ for $0 < x < 1$ if $\theta = 2$.