

所別：數學系碩士班 不分組 科目：微分方程

1.(20%) Find the value of b for which the following equation

$$(xy^2 + bx^2y)dx + (x + y)x^2dy = 0$$

is exact, and then solve it using that value of b .

2.(20%) By the method of variation of parameters show that the solution of the initial value problem

$$y'' + 2y' + 2y = f(t); \quad y(0) = 0, \quad y'(0) = 0$$

$$\text{is } y = \int_0^t e^{-(t-\tau)} f(\tau) \sin(t - \tau) d\tau.$$

3.(20%) Find a particular solution of

$$y'' - 3y' - 4y = 3e^{2t} + 2\sin t - 8e^t \cos 2t.$$

4.(20%) Find the general solution of the system

$$\frac{dx_1}{dt} = -2x_1 + x_2 + 2e^{-t},$$

$$\frac{dx_2}{dt} = x_1 - 2x_2 + 3t.$$

5.(20%) Find the solution of the differential equation

$$2y'' + y' + 2y = g(t), \quad y(0) = 0, \quad y'(0) = 0,$$

where

$$g(t) = u_5(t) - u_{20}(t) = \begin{cases} 1, & 5 \leq t < 20, \\ 0, & 0 \leq t < 5 \text{ and } t \geq 20. \end{cases}$$

