

# 國立中央大學九十一年度碩士班研究生入學試題卷

所別: 數學系 不分組 科目: 離散數學 共 1 頁 第 1 頁

1. (20%) Let  $k$  be any positive integer.

$$\text{Prove that } \sum_{i=1}^n \binom{kn}{i} \binom{n}{i} = \binom{(k+1)n}{n}.$$

2. (20%) Show that a graph is 2-colorable if and only if it has no circuits of odd length.

3. (20%) There are  $n$  men and  $n$  women, and each man likes exactly  $k$  women and each woman likes exactly  $k$  men. Assuming that  $a$  likes  $b$  if and only if  $b$  likes  $a$ , show that the  $2n$  people can be paired off into  $n$  couples so that each man is paired with a woman he likes and who likes him.

4. (20%) Let  $r, s$  be positive integers. Show that every sequence of  $rs+1$  distinct real numbers has an increasing subsequence of size  $r+1$  or a decreasing subsequence of size  $s+1$ .

5. (20%) Construct a complete orthogonal family of Latin squares of order 4.