國立中央大學九十學年度碩士班研究生入學試題卷

所別: 數學系 不分組 科目: 線性代數 共 1 頁 第 1 頁

(5分) 1. Show that there are no 2×2 matrix A and B such that $AB - BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(10分) 2. Let V be a vector space, T is a linear map from V to V. If $T^2 = 0$, show that $I - T$ is bijective.

(15分) 3. Find the characteristic polynomial, eigenvalues and eigenvectors of T, $T(x,y,z) = (2x+y, y-z, 2y+4z)$

(15分) 4. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be given by $T(x,y,z) = (3x-z, 2y, -x+3z)$ Verify that T is self-adjoint and put it into diagonal form.

(15分) 5. Use the principal axis to sketch the graph of the equation $5x^2 - 6xy + 5y^2 - 24\sqrt{2}x + 8\sqrt{2}y + 56 = 0$.

(15分) 6(a) If A is an $m\times n$ matrix, show that $\text{rank}(A) = \text{rank}(A^T)$

(b) If A and B are $m\times n$ matrices, show that $\text{rank}(A+B) \le \text{rank}(A) + \text{rank}(B)$

(c) If A is an $m\times n$ matrix, B is an $n\times s$ matrix, then $\text{rank}(AB) \le \min(\text{rank}(A), \text{rank}(B))$

(15分) 7. Let $A = \begin{bmatrix} 1 & 1 & 3 & 2 \\ -2 & -1 & 2 & 1 \\ 7 & 4 & -3 & -1 \\ 0 & 0 & 5 & 3 \end{bmatrix}$

(a) Find a basis for the row space of A consisting of vectors not row vectors of A.

(b) Find a basis for the row space of A consisting of vectors that are row vectors of A.

(c) Find a basis for the null space of A

(10分) 8. Let M, N, P be linear subspaces of a vector space V. Show that if $P \supset M$, then $P \cap (M+N) = M + (P \cap N)$.