中央大學八十九學年度碩士班研究生入學試題卷

1. (20%) Let X and Y be independent random variables having the common $N(0,1)$ distribution. (i) Find the distribution of $X - X$. (ii) Find the distribution of $X - Y$.

2. (30%) Let $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$ denote three estimators of θ and

$$\hat{\theta}_1 \leq \hat{\theta}_2 \leq \hat{\theta}_3$$

almost surely. (i) If both $\hat{\theta}_1$ and $\hat{\theta}_3$ are consistent, is $\hat{\theta}_2$ also consistent ? (ii) If both $\hat{\theta}_1$ and $\hat{\theta}_3$ are sufficient for θ, is $\hat{\theta}_2$ also sufficient ? (iii) If both $\hat{\theta}_1$ and $\hat{\theta}_3$ are efficient, is $\hat{\theta}_2$ also efficient ?

3. (20%) Let $\{X_i, i = 1, \cdots, n\}$ denote a sample and $\{Y_i = aX_i + b, i = 1, \cdots, n\}$, where a and b are nonzero constants. (i) Are the sample variances of $\{X_i, i = 1, \cdots, n\}$ and $\{Y_i, i = 1, \cdots, n\}$ equal ? Explain. (ii) Find the sample correlation coefficient of $\{(X_i, Y_i), i = 1, \cdots, n\}$.

4. (20%) (i) Explain why central limit theorems are useful in statistical inference ? (ii) Explain why laws of large numbers are useful in statistical inference ?

5. (10%) Explain the following items: (i) Simple Linear Regression. (ii) Analysis of Variance. (iii) Most Powerful Test. (iv) Maximum Likelihood Estimator. (v) Confidence Interval.