20% 1. Let C denote the circle $\{z:|z|=1\}$, taken counterclockwise, show that

$$\int_C \exp(z+z^{-1})\,dz = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!}.$$

20% 2. Show that if a function f is analytic in a deleted neighborhood of a point z_0 and if z_0 is an accumulation point of zeros of f, then either z_0 is an essential singularity of f or else f is identically equal to zero.

20% 3. Evaluate the following integrals

(1) $\int_0^{\infty} \frac{\ln x}{(x^2+4)^2}\,dx$ (2) $\int_0^{\infty} \frac{1}{(x^2+1)\sqrt{x}}\,dx$

20% 4. Let f be an entire function so that $|f(z)| \le A|z|$ for all z, where A is a fixed positive number. Show that $f(z) = a_1 z$ where a_1 is a complex constant.

20% 5. Find a conformal mapping of the infinite strip $\left\{z:|\operatorname{Im} z| < \frac{\pi}{2}\right\}$ onto the disk $\{z:|z|<1\}$.