參考用

(20%) 1. (a) Suppose $X_1, X_2, \cdots, X_n$ have $P(X_i = 1) = p$, $P(X_i = 0) = 1 - p$, and $P(X_i = 1, X_j = 1) = q$ whenever $i \neq j$. Find $Var(X_1 + \cdots + X_n)$.

(b) Let $X_1, \cdots, X_n$ be independent and identically distributed random variables have variance $\sigma^2 < \infty$. Show that

$$Cov(X_i - \overline{X}, \overline{X}) = 0.$$

(20%) 2. Let the moment-generating function $M(t)$ of X exist for $-h < t < h$. Let $R(t) = \log_e M(t)$.

(a) Show that $EX = R'(0)$ and $Var X = R''(0)$.

(b) If $M(t) = (\frac{1}{4} + \frac{3e^t}{4})^{12}$, find EY^3 and $P(Y \geq 10)$.

(20%) 3. Bob flips 3 coins and Betty flips 2. Bob wins if the number of Heads he gets is more than the number Betty gets. What is the probability Bob will win?

(20%) 4. Let X_1, X_2, X_3 have joint density $f(x_1, x_2, x_3) = \frac{6}{(1+x_1+x_2+x_3)^4}$ $x_1, x_2, x_3 > 0$. Find the density function of $Y = X_1 + X_2 + X_3$ and $P(X_1 < X_2 < X_3)$.

(20%) 5. Let $X_1, X_2, \cdots, X_{20}$ be independent Poisson random variables with mean 1.

(a) State the Markov inequality, and use it to obtain a bound on $P\{\sum_{i=1}^{20} X_i > 25\}$.

(b) State the central limit theorem, and use it to approximate $P\{\sum_{i=1}^{20} X_i > 25\}$. (Express it in terms of $\Phi(x)$, where $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$.)