國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 數學研究所 不分組 科目: 高等微積分 共 1 頁 第 1 頁

All problems are equally weighted

1. Let $S = (2,3] \cup [5,7]$ be a subset of $\mathbb{R}$. Define a function $f : S \mapsto \{0,1\}$ by

$$f(x) = \begin{cases} 0 & \text{if } x \in (2,3] \\ 1 & \text{if } x \in [5,7]. \end{cases}$$

   Is f continuous on S? Why?

2. Let d be a metric on a set M. Define $\rho(x,y) = \frac{d(x,y)}{1+d(x,y)}$, for $x, y \in M$. Prove or disprove that ρ is a metric on M.

3. Prove or disprove that $f(x) = 1/(x^2+1)$ is uniformly continuous on $\mathbb{R}$.

4. Let $A \subseteq \mathbb{R}^n$ be compact. Prove or disprove that A has a countable dense subset.

5. Each f_n is differentiable on $[-1,1]$ and $f_n \to f$ uniformly on $[-1,1]$. Prove or disprove that f is also differentiable on $[-1,1]$.

6. Suppose $\sum_{n=0}^{\infty} c_n x^n$ converges at $x = R$, and $0 < \varepsilon < R$. Show that $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-R+\varepsilon, R-\epsilon]$.

7. Let $f = f(x,y)$, where $x = r\cos\theta$ and $y = r\sin\theta$. Prove that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}.$$