

# 國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 數學研究所 不分組 科目:

複變函數論

共 1 頁 第 1 頁

參考用

- 15% 1. Find a conformal mapping  $f$  of the unit disc  $\Delta = \{z: |z| < 1\}$  onto the upper half-plane with  $f(0) = i$ .
- 10% 2. Suppose  $f(z)$  is an entire function with  $|f(z)| \leq |e^z|$  for all  $z$ . Prove that  $f(z) = Ke^z$  for some constant  $K$ .
- 10% 3. Suppose that  $f(z)$  and  $g(z)$  are analytic in a region  $\Omega$  and suppose that  $\frac{f'(z_n)}{f(z_n)} = \frac{g'(z_n)}{g(z_n)}$  at a sequence of points  $\{z_n\}$  converging to a point  $z_0$  in  $\Omega$ . Show that  $f(z) = Kg(z)$  in  $\Omega$  for some constant  $K$ .
- 15% 4. Show that all the roots of  $P(z) = z^5 + 6z^3 + 2z + 10$  lie in  $\{1 < |z| < 3\}$ .
- 15% 5. For the function  $f(z) = 1/(1+z^4)$ , find the principal part of the Laurent expansion valid in a deleted neighborhood of  $z = e^{i\pi/4}$ .
- 15% 6. If  $C$  is a simple closed curve enclosing the origin, evaluate

$$\int_C z^2 \sin \frac{1}{z} dz.$$

- 20% 7. Evaluate the following

$$(a) \int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$$

$$(b) \int_0^{\infty} \frac{1}{1+x^3} dx.$$