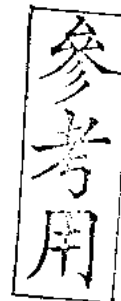


國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 數學研究所 不分組 科目: 數值分析 共 1 頁 第 1 頁

(考題共六題, 請任選五題作答)



1. (a) (10 分) Derive the rate-of-convergence of Newton's method for finding the root of the equation $f(x) = 0$
- (b) (10 分) Derive directly the formula of Newton's method for finding the roots of system of two equations $f(x, y) = 0$ and $g(x, y) = 0$.

2. For data of 10 points, $(x_i, y_i), i \in 1, \dots, 10$,

- (a) (5 分) What are the conditions required for using cubic spline interpolation functions to interpolate the data ?
- (b) (15 分) if we use spline to make the interpolation and want to let the first derivative and the second derivative to be continuous at those points, can we enforce those conditions by using linear splines, quadratic splines, or cubic splines? Give your reason.

3. (a) (10 分) For an initial-value problem, $y' = f(x, y)$, with initial condition, $y(x_i) = y_i$, show that the midpoint Euler method has a local truncation error of $O(h^3)$. i.e. $y(x_{i+1}) = y_{i+1} + O(h^3)$
- (b) (10 分) Using midpoint Euler method to iterate one step with step size $h = 0.1$ for the following initial-value problem,

$$y'' + 2y' - 3y = 0 \quad y(0) = 1, \quad y'(0) = 0$$

4. (20 分) If a quadrature rule can be written as follow,

$$\int_{-1}^1 f(x) dx = w_0 f(-1) + w_1 f(1) + w_2 f'(-1) + w_3 f'(1)$$

determine w_0, w_1, w_2, w_3

5. (a) (10 分) For system $Ax = b$ where A is a SPD, if δb is a perturbation of b and δx is the resulting perturbation, show that

$$\frac{\|\delta x\|}{\|x\|} \leq C \frac{\|\delta b\|}{\|b\|}$$

where C is the condition number of A

(b) (10 分) If we define norm of A is

$$\|A\| = \max_{x \neq 0} \frac{\|Ax\|}{\|x\|}$$

then show that

$$\frac{\|\delta x\|}{\|x + \delta x\|} \leq C \frac{\|\delta A\|}{\|A\|}$$

6. Verify that the Gaussian-elimination to solve a system of N linear equations for N unknowns, i.e. $Ax = b$ requires

- (a) (6 分) $\frac{N(N+1)}{2}$ division
- (b) (7 分) $\frac{N(N-1)(2N+5)}{6}$ multiplication
- (c) (7 分) $\frac{N(N-1)(2N+5)}{6}$ additions/subtractions

operations respectively